Traffic Load in a Dense Wireless Multihop Network*

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ABSTRACT

In wireless multihop networks each node acts as a relay for the other nodes. Consequently, the distribution of the traffic load has a strong spatial dependence. We consider a dense multihop network where the routes are approximately straight line segments. To this end we introduce the so-called line segment traversing process which defines the movement of points in a given region. In particular, the points move along the line segments with a spatial velocity which depends on the current location of the point. We use this process to model the movement of packets and utilise its properties to study the relayed traffic load which corresponds to the traffic load experienced by a node in a given location, and to study the queueing delays as a function of the location using the spatial velocity of the line segment process. The efficiency of a wireless multihop network depends significantly on the used MAC protocol, which then has an impact on queueing delays in a congested network. Our model can be adapted to any given MAC protocol by a proper choice of the spatial velocity. Additionally, from the model we also obtain an expression for the mean one-way delay in the network, which is itself an important performance measure of the network. Finally, we use ns2-simulations to validate some of the key ideas, along with several numerical examples illustrating the effects of MAC protocols on the mean end-to-end delay and power (ratio of throughtput to mean delay).

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*Wireless communication*; C.4 [**Performance Systems**]: Modeling techniques; G.3 [**Probability and Statistics**]: Stochastic processes

General Terms

Performance, Design

Keywords

Wireless Multihop Network, Traffic Load, Adhoc Network

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1. INTRODUCTION

The forwarding of data packets in wireless multihop networks relies on the nodes (i.e., the client devices) taking an active part in the traffic forwarding process. Examples of these include sensor networks with typically stationary nodes and MANETs with mobile nodes. This means that any given node, in addition to generating its own new traffic, acts as a relay (router) for the traffic originating from other nodes in the network. In practise, most of the traffic going through a node in a multihop network may be relayed traffic. To relay the traffic the nodes need information about their respective neighbours to decide where to relay the traffic for a given destination. Determining the appropriate neighbours is the task of the routing protocol, of which AODV is one practical example. A fundamental notion related to the traffic performance of such a network is the distribution of the traffic load in the network. The traffic load distribution depends heavily on the routing protocol (i.e., how the routes are chosen), the possible mobility pattern of the users, as well as, the traffic pattern.

In this paper, we derive a model for the traffic load distribution in a dense wireless multihop network. We concentrate on the case where the routing protocol performs shortest path routing. This, combined with the assumption of a dense network, implies that the routes which the packets traverse are essentially straight line segments. Then the transmission of packets constitutes a line process on a plane, where the end points of the lines correspond to the sources and the destinations of packet transmissions. The distribution of the end points in the model can be arbitrary. We refer to this process as the *line segment traversing process*. Furthermore, a point (packet) in the line segment traversing process may have a spatial velocity that depends on the location of the point. Under these assumptions the distribution for the location of a single packet can be computed by evaluating a certain integral expression. Moreover, we are able to determine a mean arrival rate of packets across an arbitrary curve, which we will utilise to model the arrival of packets into the proximity of a node.

We use the line segment traversing process to study several phenomena related to traffic load in ad hoc networks. One deals with the relayed traffic load (spatial frequency of successful transmissions), which corresponds to the pdf of the packet location. For example, if the transmission range is so large that all nodes hear each other, the locations of packet transmissions correspond to locations of the nodes, i.e., the stationary node distribution. When multihop routes are used (with a sufficiently large transmission range and number of nodes n) we obtain, due to the traffic relaying, a distribution for the locations of packet transmissions that is more concentrated into the center of the area. Using the line segment traversing process we present expressions to evaluate the rise in the frequency of transmissions in different areas. The results are validated through ns2 simulations showing excellent agreement.

Additionally we present a microscopic model for studying the traffic load experienced by a single node, defined as the mean number of transmissions per unit time within the node's proximity. The analysis utilises the results on the line segment traversing process presented in this paper. Treating a single node as a queue with a given arrival rate, the stability of the queue can be analyzed. By employing certain approximations, the familiar $O(1/\sqrt{n})$ scaling law for the capacity of ad hoc networks can be derived.

Finally, our model can take into account queueing delays along a multihop path. This is done by approximating the time it takes for a packet to travel to the destination as resulting from traversing a path on which the velocity changes continuously in proportion to the load along the path. To this end, we apply the spatial node velocity component in the line segment traversing process, which allows us to model any given MAC protocol. We give an explicit formula for average end-to-end delay in the network and study also the tradeoff between the delay and throughput.

The rest of the paper is organised as follows. Section 2 introduces the line segment traversing process. In Section 3 we state our assumptions. Section 4 analyses the relayed traffic load, and Section 5 queueing delays. In Section 6 we present some numerical examples, and finally, Section 7 concludes the paper.

1.1 Related work

Our work is related to the work by Pham et al. in [7] and [8], The main idea in these is to analyze the benefits of using multipath routing instead of just single shortest path routing. To this end a model for the packet transmissions is presented assuming that the routes that the packets take are essentially straight lines (similarly as we do) and that the senders and receivers are uniformly distributed. Additionally, in [8], the model is extended to include a delay analysis of the network under shortest path routing and multipath routing. In [7] and [8] the mean number of packets the node itself has to transmit is studied, i.e., the quantity defining the traffic load is slightly different than in this paper. However, our objective is to study the traffic seen by a node at a given location, which is important, e.g., when one wants to determine the maximum sustainable load a given network can handle. Additionally, the assumption in the delay analysis of [8] is that a node at a given location can be modelled as a finite M/M/1 queue with an arrival rate depending on the location, which may be a valid model for some particular MAC protocol. However, our approach allows modeling of various types of MAC protocols through the notion of the spatial velocity. Also, our analysis is not restricted by the assumption of uniformly distributed senders and receivers (which allows us to, e.g., study the impact of mobility), and we present an exact analysis of the process describing the packet movement, whereas the results in [7] and [8] are approximate in nature. In [2] the model from [7] is extended to more accurately characterize the impact of multi-path routing.

2. PRELIMINARIES

Here we give a formal definition for a process, which generates line segments into a plane at random time instants. Using this process we define a point movement process, where each line segment creates a point which moves from one end of a line segment to the other end at a certain speed and then disappears. These points are used to model the movement of a packet through the network.

Definition 1 (line segment process) *A process is said to be a time homogeneous line segment process in $\mathcal{A} \subset \mathbb{R}^2$ with intensity $\lambda(\mathbf{r}_1, \mathbf{r}_2) : (\mathcal{A}, \mathcal{A}) \to \mathbb{R}$ if the number of line segments generated from a differential area element at $\mathbf{r}_1 \in \mathcal{A}$ to a differential area element at $\mathbf{r}_2 \in \mathcal{A}$ constitutes a Poisson process with intensity of $\lambda(\mathbf{r}_1, \mathbf{r}_2) \cdot dA^2$.*

In other words, a line segment from a differential area element dA about $\mathbf{r}_1 \in \mathcal{A}$ to a differential area element dA about $\mathbf{r}_2 \in \mathcal{A}$ is generated during a short time interval Δt with a probability equal to $\lambda(\mathbf{r}_1, \mathbf{r}_2) \cdot dA^2 \cdot \Delta t$. Note that, the line segment process is, in fact, a special case of a Poisson point process in $\mathbb{R}^4$. A realisation of this line process is defined by an infinite sequence of triples

$$(P_0^{(s)}, P_0^{(d)}, t_0),\ (P_1^{(s)}, P_1^{(d)}, t_1),\ \ldots,$$

where the $P_i^{(s)}$ and $P_i^{(d)}$ correspond to the source and destination points of the line segments, and the t_i to the time instants.

Remark 2 *In the line segment process new line segments are generated according to a Poisson process with a rate given by*

$$\Lambda = \int_{\mathcal{A}} \int_{\mathcal{A}} \lambda(\mathbf{r}_1, \mathbf{r}_2)\ d^2\mathbf{r}_2\ d^2\mathbf{r}_1. \tag{1}$$

Corollary 3 *The mean length of a line segment is*

$$\overline{\ell} = \frac{1}{\Lambda} \int_{\mathcal{A}} \int_{\mathcal{A}} |\mathbf{r}_2 - \mathbf{r}_1| \cdot \lambda(\mathbf{r}_1, \mathbf{r}_2)\ d^2\mathbf{r}_2\ d^2\mathbf{r}_1. \tag{2}$$

2.1 Point transitions along the line segments

Consider next a process where each line segment creates a point which moves from the source to the destination at a certain speed.

Definition 4 (line segment traversing process) *A process where each line segment $\ell_i = (\mathbf{r}_1, \mathbf{r}_2)$ generated by a time homogeneous line segment process triggers a point to move from the source $\mathbf{r}_1$ to the destination $\mathbf{r}_2$ is said to be a line segment traversing process if the velocity of the point at $\mathbf{r} \in \ell_i$ is given by*

$$v_i(\mathbf{r}) = v_i \cdot \nu(\mathbf{r}),$$

where the $v_i \sim v$ are i.i.d. nominal velocities (random variables) and $\nu(\mathbf{r}) : \mathcal{A} \to \mathbb{R}$ corresponds to the local spatial velocity (function of location).

Corollary 5 *The mean transition time along a random line, denoted by $\mathrm{E}[T]$, is given by*

$$\mathrm{E}[T] = \mathrm{E}[\mathrm{E}[T \mid v]] = \mathrm{E}[1/v] \cdot \mathrm{E}[T^*],$$

where $\mathrm{E}[T^*]$ denotes the mean transition time conditioned on that the nominal velocity v is a constant, $v = 1$,

$$\mathrm{E}[T^*] = \frac{1}{\Lambda} \int_{\mathcal{A}} \int_{\mathcal{A}} |\mathbf{r}_2 - \mathbf{r}_1|\, \lambda(\mathbf{r}_1, \mathbf{r}_2) \cdot \overline{\nu}(\mathbf{r}_1, \mathbf{r}_2)\ d^2\mathbf{r}_2\ d^2\mathbf{r}_1, \tag{3}$$

and $\overline{\nu}(\mathbf{r}_1, \mathbf{r}_2)$ is the mean velocity on transition $(\mathbf{r}_1, \mathbf{r}_2)$ conditioned on that $v = 1$,

$$\overline{\nu}(\mathbf{r}_1, \mathbf{r}_2) = \int_0^1 [\nu(h\mathbf{r}_2 + (1-h)\mathbf{r}_1)]^{-1}\ dh.$$

Remark 6 *For $\nu(\mathbf{r}) = 1$ we have $\mathrm{E}[T^*] = \overline{\ell}$.*

Remark 7 *The mean number of moving points in a system, denoted by $\mathrm{E}[n]$, is given by (Little's result)*

$$\mathrm{E}[n] = \Lambda \cdot \mathrm{E}[T]. \tag{4}$$

2.2 Distribution of moving points

One important property of a point movement process is the density of points at a given location. For the line segment traversing process we have the following result.

Proposition 8 *Pdf of the location of a point at an arbitrary time instant is given by*

$$f(\mathbf{r}) = \frac{1}{\mathrm{E}[T^*]\cdot\Lambda\cdot\nu(\mathbf{r})}\int_0^{2\pi}\int_0^{a(\phi+\pi)}\int_0^{a(\phi)}(r_1+r_2)\cdot \lambda(\mathbf{r}_1,\mathbf{r}_2)\,dr_2\,dr_1\,d\phi, \tag{5}$$

where $\mathbf{r}_1 = \mathbf{r} + r_1\cdot(\cos(\phi+\pi),\,\sin(\phi+\pi))$*, and* $\mathbf{r}_2 = \mathbf{r} + r_2\cdot(\cos(\phi),\,\sin(\phi))$.

PROOF. The probability of finding a point inside a small area element dA about $\mathbf{r}\in\mathcal{A}$ at a random time instant is proportional to $\mathrm{E}[\ell\cap dA]$ where ℓ denotes an arbitrary line segment. Thus,

$$\begin{aligned} f(\mathbf{r}) &= \frac{\mathrm{E}[(\ell\cap dA)/(v\cdot\nu(\mathbf{r}))]}{\mathrm{E}[T]\cdot dA} \\ &= \frac{\mathrm{E}[(\ell\cap dA)]\cdot\mathrm{E}[1/v]\cdot 1/\nu(\mathbf{r})}{\mathrm{E}[1/v]\cdot\mathrm{E}[T^*]\cdot dA} = \frac{\mathrm{E}[(\ell\cap dA)]}{\mathrm{E}[T^*]\cdot dA\cdot\nu(\mathbf{r})}. \end{aligned}$$

Let $g_s(\mathbf{r}_1)$ denote the probability that a line segment starts from $\mathbf{r}_1$,

$$g_s(\mathbf{r}_1) = \frac{1}{\Lambda}\int_{\mathcal{A}}\lambda(\mathbf{r}_1,\mathbf{r}_2)\;d^2\mathbf{r}_2 = \frac{\lambda_s(\mathbf{r}_1)}{\Lambda},$$

where $\lambda_s(\mathbf{r}_1)$ denotes the generation rate of lines starting from $\mathbf{r}_1$ per unit time and unit area. Thus, the pdf of an arbitrary point is

$$\begin{aligned} f(\mathbf{r}) &= \frac{1}{\mathrm{E}[T^*]\cdot dA\cdot\nu(\mathbf{r})}\int_{\mathcal{A}} g_s(\mathbf{r}_1)\cdot\mathrm{E}[\ell\cap dA\,|\,\mathbf{r}_1]\;d^2\mathbf{r}_1 \\ &= \frac{1}{\mathrm{E}[T^*]\cdot dA\cdot\Lambda\cdot\nu(\mathbf{r})}\cdot \int_0^{2\pi}\int_0^{a(\phi+\pi)} r_1\cdot\lambda_s(\mathbf{r}_1)\cdot\mathrm{E}[\ell\cap dA\,|\,\mathbf{r}_1]\;dr_1\,d\phi, \end{aligned}$$

where $\mathbf{r}_1 = \mathbf{r} + r_1\cdot(\cos(\phi+\pi),\,\sin(\phi+\pi))$, and $a(\phi)$ is the distance from point $\mathbf{r}$ to the boundary in direction ϕ. Let $dA = \Delta\cdot r_1\,d\phi$. For the conditional expectation we have (see Figure 1),

$$\begin{aligned} \mathrm{E}[\ell\cap dA\,|\,\mathbf{r}_1] &= \Delta\int_0^{a(\phi)}(r_1+r_2)\cdot\frac{\lambda(\mathbf{r}_1,\mathbf{r}_2)}{\lambda_s(\mathbf{r}_1)}\,d\phi\,dr_2 \\ &= \frac{dA}{r_1\cdot\lambda_s(\mathbf{r}_1)}\int_0^{a(\phi)}(r_1+r_2)\cdot\lambda(\mathbf{r}_1,\mathbf{r}_2)\,dr_2, \end{aligned}$$

where $\mathbf{r}_2 = \mathbf{r} + r_2\cdot(\cos(\phi),\,\sin(\phi))$. Hence,

$$f(\mathbf{r}) = \frac{1}{\mathrm{E}[T^*]\cdot dA\cdot\Lambda\cdot\nu(\mathbf{r})}\int_0^{2\pi}\int_0^{a(\phi+\pi)} r_1\cdot\lambda_s(\mathbf{r}_1)\cdot \frac{dA}{r_1\cdot\lambda_s(\mathbf{r}_1)}\int_0^{a(\phi)}(r_1+r_2)\cdot\lambda(\mathbf{r}_1,\mathbf{r}_2)\,dr_2\,dr_1\,d\phi,$$

and cancelling the common terms completes the proof. □

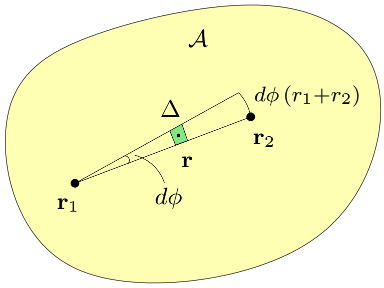


Figure 1: Notation for the line segment traversing process pdf.

Corollary 9 *Multiplying* (5) *by* $\mathrm{E}[n]$*, given by* (4)*, yields the density of points at* $\mathbf{r}$*,*

$$n(\mathbf{r}) = \frac{\mathrm{E}[1/v]}{\nu(\mathbf{r})}\int_0^{2\pi}\int_0^{a(\phi+\pi)}\int_0^{a(\phi)}(r_1{+}r_2)\lambda(\mathbf{r}_1,\mathbf{r}_2)\,dr_2\,dr_1\,d\phi. \tag{6}$$

Remark 10 *Generally, for an arbitrary line segment process it holds that*

$$f(\mathbf{r}) = \frac{\overline{\ell}}{\mathrm{E}[T^*]}\cdot\frac{f_0(\mathbf{r})}{\nu(\mathbf{r})}, \tag{7}$$

where $f(\mathbf{r})$ *is the stationary distribution of the complete line segment traversing process with an arbitrary* $\nu(\mathbf{r})$*, and* $f_0(\mathbf{r})$ *is the stationary distribution of the respective process with* $\nu(\mathbf{r}) = 1$*.*

Example 11 *Random waypoint (RWP) process is a commonly used mobility model (see [1, 6, 3, 4]), where each user moves along a zigzag line from one waypoint to the next. The waypoints are distributed according to some pdf* $g(\mathbf{r})$ *(typically uniform). Let* $\lambda(\mathbf{r}_1,\mathbf{r}_2) = \Lambda\cdot g(\mathbf{r}_1)\cdot g(\mathbf{r}_2)$*, i.e., the source and the destination points are i.i.d. random variables. It turns out that the stationary distribution of the line segment traversing process is identical to that of the corresponding RWP process with a waypoint distribution* $g(\mathbf{r})$*. Substituting that into* (5) *gives*

$$f(\mathbf{r}) = \frac{1}{\mathrm{E}[T^*]\cdot\nu(\mathbf{r})}\int_0^{2\pi}\int_0^{a(\phi+\pi)}\int_0^{a(\phi)}(r_1+r_2)\cdot g(\mathbf{r}_1)\cdot g(\mathbf{r}_2)\cdot\;dr_2\,dr_1\,d\phi. \tag{8}$$

With $\nu(\mathbf{r}) = 1$ *we have* $\mathrm{E}[T^*] = \overline{\ell}$ *and* (8) *can be written as*

$$f_0(\mathbf{r}) = \frac{1}{\overline{\ell}}\int_0^{2\pi}\int_0^{a(\phi+\pi)}\int_0^{a(\phi)}(r_1{+}r_2)\cdot g(\mathbf{r}_1)\cdot g(\mathbf{r}_2)\,dr_2\,dr_1\,d\phi, \tag{9}$$

which is in agreement with the expression for the pdf of the non-uniform RWP process from [3]. Moreover, according to Little's result $\mathrm{E}[n] = \Lambda\cdot\overline{\ell}$*, and*

$$n_0(\mathbf{r}) = \Lambda\cdot\overline{\ell}\cdot f_0(\mathbf{r}).$$

With $g(\mathbf{r}) = 1/A$*, where* A *is the area of the domain, we obtain*

$$f(\mathbf{r}) = \frac{1}{\mathrm{E}[T^*]\cdot\nu(\mathbf{r})}\int_0^{2\pi} a(\phi)\cdot a(\phi+\pi)\,[a(\phi)+a(\phi+\pi)]\;d\phi,$$

i.e., the pdf of the point in the traditional RWP process with a uniform waypoint distribution [3].

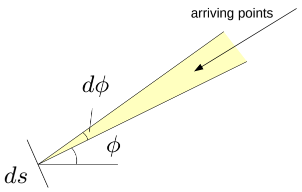


Figure 2: Angular flux.

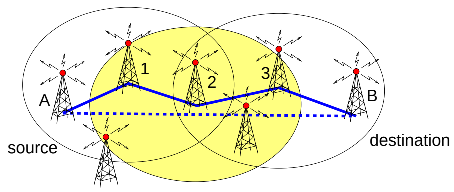


Figure 3: Shortest path and its approximation by line segment.

2.3 Mean flow of points

Another important property resulting from the movement is the rate at which points cross a given boundary. This property is later used to model the arrival rate of packets into the transmission range of a network node. To this end we first define the so-called scalar flux which represents the flow of points at a given location:

Definition 12 (angular flux) *Angular flux at point* $\mathbf{r}$ *in direction* ϕ*, denoted by* $\psi(\mathbf{r}, \phi)$*, is the rate at which points moving in the direction* $(\phi, \phi + d\phi)$ *cross a differential line segment at point* $\mathbf{r}$ *perpendicular to* ϕ *per unit time per unit length (see Figure 2).*

Definition 13 (scalar flux) *Scalar flux at point* $\mathbf{r}$ *is given by*

$$\Phi(\mathbf{r}) = \int_0^{2\pi} \psi(\mathbf{r}, \phi)\, d\phi. \tag{10}$$

Corollary 14 *From* (10) *one obtains an identity for the scalar flux,*

$$\Phi(\mathbf{r}) = \lim_{d \to 0} \frac{q(\mathbf{r}, d)}{2d}, \tag{11}$$

where $q(\mathbf{r}, d)$ *denotes the arrival rate of points (or packets) into a disk at* $\mathbf{r}$ *with radius* d*.*

Proposition 15 *For the scalar flux* $\Phi(\mathbf{r})$ *at* $\mathbf{r}$ *with a constant nominal velocity* $v = 1$ *it holds that*

$$\Phi(\mathbf{r}) = n(\mathbf{r}) \cdot \nu(\mathbf{r}). \tag{12}$$

Proposition 16 *The rate of points crossing a given curve* $\mathcal{C}$ *in the direction of normal* $\mathbf{n}(s)$ *is given by*

$$q(\mathcal{C}) = \int_{\mathcal{C}} \int_{-\pi/2}^{\pi/2} \cos\phi \cdot \psi(\mathbf{r}(s), \theta_n(s) + \phi)\, d\phi\, ds, \tag{13}$$

where $\theta_n(s)$ *denotes the direction of the normal and* $\psi(\mathbf{r}, \phi)$ *is the angular flux at point* $\mathbf{r}$ *in direction* ϕ*, for which it holds that*

$$\psi(\mathbf{r}, \phi) = \int_0^{a(\phi+\pi)} \int_0^{a(\phi)} (r_1 + r_2) \cdot \lambda(\mathbf{r}_1, \mathbf{r}_2)\, dr_2\, dr_1, \tag{14}$$

where $\mathbf{r}_1 = \mathbf{r} + r_1 \cdot (\cos(\phi + \pi), \sin(\phi + \pi))$ *and* $\mathbf{r}_2 = \mathbf{r} + r_2 \cdot (\cos(\phi), \sin(\phi))$.

PROOF. The choice of velocity clearly has no effect on the arrival rate across a given boundary as long as the mean transition time is finite. Hence, without loss of generality we can assume a unit nominal velocity, $v = 1$, so that the velocity of a point at $\mathbf{r}$ is $\nu(\mathbf{r})$. Combining (6) and (12) one can identify that the quantity

$$\psi(\mathbf{r}, \phi) = \int_0^{a(\phi+\pi)} \int_0^{a(\phi)} (r_1 + r_2) \cdot \lambda(\mathbf{r}_1, \mathbf{r}_2)\, dr_2\, dr_1,$$

is the angular flux at point $\mathbf{r}$ in direction ϕ, i.e., the expected rate of crossings across a differential line segment perpendicular to direction ϕ per unit length and per unit angle. Hence, the quantity,

$$\int_{-\pi/2}^{\pi/2} \cos\phi \cdot \psi(\mathbf{r}, \theta + \phi)\, d\phi,$$

is the flux per unit length across a differential line segment at $\mathbf{r}$ having a normal pointing to the direction θ, and the flux crossing a curve $\mathcal{C}$ in the direction defined by the normal $\mathbf{n}(s)$ equals

$$q(\mathcal{C}) = \int_{\mathcal{C}} \int_{-\pi/2}^{\pi/2} \cos\phi \cdot \psi(\mathbf{r}(s), \theta_n(s) + \phi)\, d\phi\, ds,$$

where $\theta_n(s)$ is the direction of the normal at point s in the curve. □

Remark 17 *For a closed curve* $\mathcal{C}$ *the total flux from outside to inside is given by the contour integral*

$$q(\mathcal{C}) = \oint_{\mathcal{C}} \int_{-\pi/2}^{\pi/2} \psi(\mathbf{r}(s), \theta_n(s) + \phi)\, d\phi\, ds, \tag{15}$$

where $\theta_n(s)$ *is the direction of the normal* $\mathbf{n}(s)$ *which points to inside direction of the curve.*

Remark 18 *Using the angular flux defined in* (14) *the point density, given by* (6)*, can be written as*

$$n(\mathbf{r}) = \frac{\mathrm{E}[1/v]}{\nu(\mathbf{r})} \int_0^{2\pi} \psi(\mathbf{r}, \phi)\, d\phi.$$

Example 19 *For a uniform line segment generation rate,* $\lambda(\mathbf{r}_1, \mathbf{r}_2) = \lambda$*, the angular flux reduces into*

$$\psi(\mathbf{r}, \phi) = \frac{\lambda}{2} \cdot a(\phi) \cdot a(\phi + \pi) \cdot [a(\phi) + a(\phi + \pi)],$$

which is in agreement with [4] (with $\lambda = 1/\bar{\ell}$*).*

3. MODEL FOR THE PACKET MOVEMENT

As already mentioned, we assume a dense network with a large number of nodes. Fig. 3 represents a typical example of a multihop transmission. Node A sends a packet to node B and three intermediate nodes, 1, 2 and 3, along the shortest route act as relays. The idea is to consider the straight line segment from A to B instead of the actual zigzag line via the nodes 1, 2 and 3. These line segments can be modelled by the line segment process introduced in Section 2. In summary, we assume (similarly as in [7]):

1. A dense multihop network with a large number of nodes, denoted by N, in a convex domain $\mathcal{A}$.

2. Location of node i, denoted by P_i, is randomly distributed according to $g(\mathbf{r})$. With mobile nodes, from the point of view of the packet transmission, the nodes are (quasi) stationary.

3. Nodes have a fixed transmission range d, i.e., they can transmit directly to the nodes within a distance d.

4. A fully connected network and high node density so that the shortest paths are approximately straight line segments.

5. Uniform traffic, nodes send packets to all other nodes at rate λ, and total rate is $\Lambda = N(N-1) \cdot \lambda$.

6. Mean packet transmission time is $1/\mu$ and the mean packet length (in bits) is B, i.e., the nominal capacity of the channel is $C_0 = B \cdot \mu = [\text{bit/s}]$.

4. RELAYED TRAFFIC LOAD

Consider first a typical case where a packet travels through the neighbourhood of a given node and the node, or one of its neighbours, acts as a relay. As a result the node will hear the same packet several times corresponding to a certain number of transmission channel reservations. The first time corresponds to the arrival of the packet into the transmission range of the node in question. The next time corresponds to the retransmission of the packet further, performed by the node itself or by one of its neighbours. Furthermore, e.g., when the node itself acts as a relay, it will hear also a third transmission when the next node along the route transmits the packet further. These events can be approximated by considering the events when the line process cuts through the respective disk, which corresponds to two to three transmissions.

Note that the destination node B only hears the packet once when the packet "arrives" into the transmission range D_B of node B. Also the source node A and the first relaying node hear two transmissions even though the packet never "arrives" into their transmission ranges. Moreover, part of the transmissions are single-hop constituting a single transmission where the corresponding line segment may not cross the boundary of the transmission range disk of the given node. However, with the assumption of large number of nodes most routes consist of several hops and the inconsistencies mentioned above can be neglected. At this point it is quite clear that the packet movement can be modelled using the line segment traversing process according to Def. 4. In particular, the independent line segment traversing process corresponding to the movement of an arbitrary packet can be characterised as follows:

1. New line segments are generated at the total rate of $\Lambda = N \cdot (N-1) \cdot \lambda$ per unit time.

2. End points of line segments are independently and randomly distributed according to some pdf $g(\mathbf{r})$.

3. Each line segment corresponds to the transmission of a single packet along a multihop path.

Hence, we define the *relayed traffic load* as the arrival rate of packets into the transmission range of a node located at $\mathbf{r}$. With the above assumptions it can be approximated in a dense multihop network by (15). With a small transmission range d compared to the whole area $\mathcal{A}$ the relayed traffic load can be estimated by the scalar flux according to (11),

$$q(\mathbf{r}, d) \approx 2d \cdot \Phi(\mathbf{r}). \tag{16}$$

Note that this quantity neglects the fact that each arriving packet that is relayed further requires one or more transmissions before it departs the transmission range of a given node. Furthermore, this quantity also excludes possible collisions.

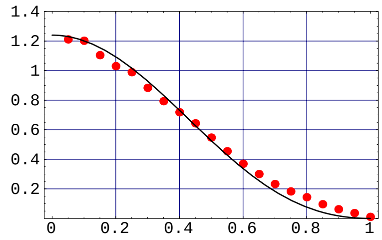


Figure 4: Normalised traffic load distribution in an wireless multihop network according to theory (solid curve) and numerical results with ns2 (dots).

4.1 Validation of the model

In order to confirm the validity of our assumptions we have simulated a standard 802.11 wireless network operating in ad hoc mode using the ns2 network simulator. In the simulation we have used $n = 40$ nodes, which move slowly in a circular area having a radius $r = 125$m according to the random waypoint model with a uniform waypoint distribution and a constant velocity of $v = 0.1$ m/s. The transmission and receive powers were adjusted so that the maximum transmission range was about $d = 50$m. Furthermore, the proactive DSDV routing protocol was used.

Our objective is to study the relayed traffic load as a function of location, i.e., how many packets are transmitted in the proximity of a given node per time unit on average. Note that relative relayed traffic load, i.e., the ratio of the rate of transmissions occurring in the proximity of node a and node b, remains the same regardless of the actual traffic load (assuming the same routes are used). To this end a low traffic load scenario was created by appropriately chosen constant bit rate (CBR) sources. In our model this scenario corresponds to a situation where the end points of the line segment process are distributed according to stationary distribution of basic RWP process (from [3]),

$$g(\mathbf{r}) \approx \frac{6(1-r^2)(27-8r^2)}{73\pi}.$$

In a low load situation we can neglect the queueing delays and the pdf of the packet location, $f(\mathbf{r})$, is easy to evaluate by numerical integration of (9). With the assumption of multihop paths, this pdf can be related to the frequency of transmissions occuring within a given location using (12) and (16), i.e., we have $\Phi(\mathbf{r}) \propto f(\mathbf{r})$.

With this in mind, we recorded from the simulations the locations of successful packet transmissions over a long time interval. From the statistics we computed the frequency of transmissions occuring within the range $(r - \Delta r, r + \Delta r)$ and divided that by $2\pi r$. In other words, we obtained an estimate for the frequency of transmissions occuring at a given distance from the origin, which according to our assumptions should be proportional to the scalar flux at a given distance. The normalised results can be seen in Figure 4 from which it is obvious that the model and numerical results are in agreement.

4.2 Capacity of the network

Note that in order to determine the capacity of the network we need to identify the location having the highest arrival rate, i.e., the bottleneck area. More specifically, we use the following definition for the capacity the multihop network:

Definition 20 (network capacity) *Capacity of the network is the maximum (uniform) packet arrival rate $\lambda_{\max}$ times the mean size*

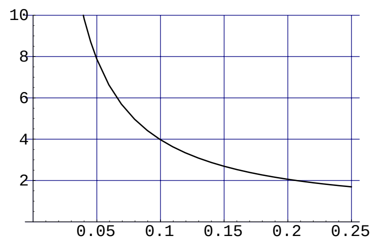


Figure 5: The maximum sustainable load $\rho^{(\mathrm{tot})}_{\max}$ in unit disk as a function of the transmission range d.

of a packet B (in bits) the network can handle,

$$C = \lambda_{\max} \cdot B = \rho_{\max} \cdot C_0.$$

Remark 21 (capacity per node) *A related performance quantity is the capacity allocated per node,*

$$C^{(1)} = \frac{C}{N}.$$

Example 22 (capacity in unit disk) *Let us assume that the nodes of a wireless multihop network are placed uniformly into a unit disk and that we want to determine the traffic load experienced by a node residing in the center of the disk as the traffic load will be clearly highest at that point. The sum of arrival and departure rates inside a concentric disk with radius d is given by (see [4])*

$$\begin{aligned}\lambda(d) &= 2 \cdot q(0, d) \\ &= \Lambda \cdot \frac{4}{\pi} \cdot d(1-d^2) \int_0^{\pi} \sin\phi \sqrt{1 - d^2\cos^2\phi}\, d\phi.\end{aligned}$$

which includes the packet flows in both directions. As explained earlier we use this as an estimate to the mean rate of transmissions occuring inside the transmission range, i.e., we assume that transmission range d is so small that the proportion of traffic having the source and destination within the transmission range of a node in the center of area is negligible. Assuming the packet lengths are i.i.d. with mean $1/\mu$ and defining the total network load as $\rho^{(\mathrm{tot})} = \Lambda/\mu$ we can determine the maximum sustainable network load as a function of d, i.e., at the point where $\lambda(d)/\mu = 1$ we have

$$\rho^{(\mathrm{tot})}_{\max} = \frac{\Lambda}{\mu} = \frac{\pi}{4d(1-d^2)\int_0^{\pi} \sin\phi \sqrt{1 - d^2\cos^2\phi}\, d\phi}.$$

For small values of d the maximum traffic load is approximately

$$\rho^{(\mathrm{tot})}_{\max} = \frac{\pi}{8d},$$

or alternatively, the capacity of the network is

$$C = \frac{\pi}{8d} \cdot C_0.$$

In Fig. 5 the maximum sustainable load $\rho^{(\mathrm{tot})}_{\max}$ is depicted as a function of the transmission range d. Note that in practice the transmission range d cannot be arbitrarily small, but instead we must ensure that the network remains fully connected most of the time. Furthermore, a small transmitting range also means that the mean number of hops each packet experiences increases considerably resulting in long end-to-end delays. Hence, the smallest practical value for d depends on the design criteria and especially on the number of nodes and their distribution in a given area.

Example 23 (scaling law) *One way to deal with the connectivity requirement is to ensure that the mean number of neighbours a typical node has, denoted by S, is high enough. Clearly,*

$$\mathrm{E}[\textit{neighbours}] = S \approx d^2 \cdot N.$$

Combining the above gives us the well-known scaling law for the capacity of the single node in a wireless multihop network,

$$C^{(1)} = \frac{C}{N} = \frac{C_0 \cdot \rho^{(\mathrm{tot})}_{\max}}{N} \approx \frac{\pi C_0}{8dN} = \frac{\pi \cdot C_0}{8\sqrt{S}} \cdot \frac{1}{\sqrt{N}},$$

i.e., the capacity of a node is proportional to $1/\sqrt{N}$.

5. QUEUEING DELAYS

In a wireless multihop network with low congestion, packets travel with an average speed of $v(\mathbf{r}) = d/\mu^{-1} = d \cdot \mu$, where d is the (mean) transmission range and μ^{-1} the mean transmission time of a packet. When congestion occurs the relaying nodes have to wait a certain time before transmitting a packet further. Let $W(\mathbf{r})$ denote the mean waiting time before a transmission at $\mathbf{r}$.

The line segment traversing process with spatial velocity distribution (see Section 2) can be applied to take into account the queueing delays. To this end, we need to find an expression for the mean waiting time $W(\mathbf{r})$ at $\mathbf{r}$ before a succesful transmission, which takes into account the local congestion and the used MAC protocol. In particular, in our model we set the nominal velocity constant, $v = 1$, and propose using

$$\nu(\mathbf{r}) = \frac{d}{W(\mathbf{r}) + 1/\mu} = \frac{d \cdot \mu}{W(\mathbf{r})\mu + 1}, \qquad (17)$$

as the spatial velocity of a packet at $\mathbf{r}$. Thus, we can compute $f(\mathbf{r})$ and $n(\mathbf{r})$ using (8) and (6), respectively. Quantity $\mathrm{E}[T] = \mathrm{E}[T^*]$, corresponds to the mean one-way delay (the line specific velocity is assumed to be constant, $v = 1$). At this point the missing part is the actual expression for the waiting time $W(\mathbf{r})$, which depends on the used MAC protocol and the traffic pattern among other things.

Denote by $f_0(\mathbf{r})$ the pdf of node location in a system with unit nominal velocity, $v = 1$, and a constant spatial velocity component, $\nu(\mathbf{r}) = 1$, corresponding to a system with no queueing delays. The mean end-to-end delay of this system is simply $\mathrm{E}[T_0] = \bar{\ell}$ and the node density is given by

$$n_0(\mathbf{r}) = \mathrm{E}[n] \cdot f_0(\mathbf{r}) = \Phi(\mathbf{r}).$$

Thus, according to (12), the packet density for a given $\nu(\mathbf{r})$ is

$$n(\mathbf{r}) = \Phi(\mathbf{r})/\nu(\mathbf{r}).$$

5.1 Mean one-way delay

One important performance measure of any network is the average end-to-end delay, i.e., the mean one-way delay, which we denote by $\mathrm{E}[T]$. For a dense wireless multihop network we can immediately write down some "asymptotic" results. More specifically, it holds that

$$\lim_{\Lambda \to 0} \mathrm{E}[T] = \bar{\ell}/(d \cdot \mu), \qquad \lim_{d \to 0} \mathrm{E}[T] = \infty.$$

Furthermore, for large enough d the system reduces into a big server which can be approximated, e.g., by an M/M/1-queue which yields

$$\mathrm{E}[T] = \frac{1}{\mu - \Lambda} \qquad \text{when } d > \operatorname{diam} \mathcal{A}$$

where $\operatorname{diam} \mathcal{A}$ denotes the largest diameter of domain $\mathcal{A}$. Generally, applying Little's result for $n(\mathbf{r})$ gives the mean one-way delay,

$$\mathrm{E}[T] = \frac{\mathrm{E}[n]}{\Lambda} = \frac{1}{\Lambda} \int_{\mathcal{A}} n(\mathbf{r})\, d^2\mathbf{r}. \qquad (18)$$

5.2 Delay-throughput tradeoff

Generally, when the traffic load increases also the mean delays increase in a network, i.e., there is a tradeoff between end-to-end delay and obtained throughput. Let us next consider a performance measure called power introduced by Kleinrock [5], which is defined to be the ratio between the throughput and the mean delay,

$$\gamma^* = \frac{\Lambda \cdot B}{\mathrm{E}[T]} = \frac{(\Lambda)^2}{\mathrm{E}[N]} \cdot B, \tag{19}$$

where B is the mean size of packet (in bits). For simplicity, instead of considering (19) we use a slightly modified version given by

$$\gamma = \gamma^* \cdot \frac{1}{\mu^2 B} = \frac{\rho^{(\mathrm{tot})}}{\mathrm{E}[T]} \cdot \frac{1}{\mu} = \frac{(\rho^{(\mathrm{tot})})^2}{\mathrm{E}[N]}, \tag{20}$$

i.e., the ratio between the total offered load, $\rho^{(\mathrm{tot})}$, times the mean transmission time, $1/\mu$ (constant), to the mean end-to-end delay, $\mathrm{E}[T]$, or alternatively, the ratio between square of the offered load $\rho^{(\mathrm{tot})}$ and the mean number of packets in the system. Note that (19) and (20) differ only by a constant factor of $\mu^2 B$. The benefit from using (20) is the fact that it gives us the same value irrespective of the chosen time units/scale. Generally, one is interested in finding the optimal traffic load $\rho^{(\mathrm{opt})}$ which maximises the power according to (20). In a wireless multihop network the mean delay depends heavily on both congestion (i.e., queueing delays) and the transmission range (i.e., the mean number of hops).

6. EXAMPLES OF DELAY MODELLING

To complete the model one needs to find an explicit expression for the spatial velocity $\nu(\mathbf{r})$ reflecting the delays due to packet contention on MAC layer. To this end one option is to assume that each node and it's neighbourhood behave approximately as a single server queue for which results are readily available. In the following, we present several idealised models for spatial velocity and illustrate the concept by numerical examples.

6.1 Elementary models for spatial velocity

6.1.1 M/M/1-server approximation

For now, without presenting any arguments, let us assume that the queueing delays in a dense multihop wireless network can be approximated by an M/M/1-queue. In particular, let us assume that a transmission range of a node acts like an M/M/1-queue with Poissonian arrivals and exponential service times. Using (13) we can compute the mean arrival rate into a disk having a center at $\mathbf{r}$ and radius d denoted by $q(\mathbf{r}, d)$. The mean service time in our case is the mean transmission time of a packet, which we have denoted by $1/\mu$. In an M/M/1-queue the mean waiting time is given by

$$\mathrm{E}[W_{\mathrm{MM1}}] = \frac{\rho}{1-\rho} \cdot \frac{1}{\mu},$$

and hence we can assume that the average velocity of a packet at point $\mathbf{r}$ could be estimated by

$$\nu(\mathbf{r}) = \frac{d \cdot \mu}{\rho/(1-\rho)+1} = (\mu - q(\mathbf{r}, d)) \cdot d,$$

where $q(\mathbf{r}, d)$ is the arrival rate of packets into a disk at $\mathbf{r}$ with radius d. For small values of d we have $q(\mathbf{r}) \approx 2d \cdot \Phi(\mathbf{r})$ and

$$\nu(\mathbf{r}) = (\mu - 2d \cdot \Phi(\mathbf{r})) \cdot d.$$

In particular, the packet density at $\mathbf{r}$ is given by

$$n(\mathbf{r}) = \frac{\Phi(\mathbf{r})}{(\mu - 2d \cdot \Phi(\mathbf{r})) \cdot d}. \tag{21}$$

Furthermore, assume that the maximum load a particular MAC protocol can handle is less than 1,

$$\rho_{\max} = 2d \cdot \Phi_{\max}/\mu.$$

Then a similar analysis as above yields

$$n(\mathbf{r}) = \frac{\Phi(\mathbf{r})}{(1 - \Phi(\mathbf{r})/\Phi_{\max}) \cdot d \cdot \mu}. \tag{22}$$

6.1.2 M/D/1-server approximation

Assuming fixed size packets, i.e., fixed time transmission periods, we can approximate the queueing time by an appropriate M/D/1-queue. Similarly as above, one obtains

$$n(\mathbf{r}) = \frac{\mu - d \cdot \Phi(\mathbf{r})}{d \cdot \mu \cdot (\mu - 2d \cdot \Phi(\mathbf{r}))} \cdot \Phi(\mathbf{r}). \tag{23}$$

6.1.3 Simplified Aloha approximation

For simplicity let us next assume a basic Aloha MAC with fixed size packets and without any collision avoidance mechanisms. In other words, each station tries to send a packet after an exponentially distributed interval and a packet is succesfully transmitted if no other transmission overlaps with the given transmission within the corresponding proximity. For this we have,

$$p = \mathrm{P}\{\text{succesful transmission}\} = e^{-\lambda \cdot 2 \cdot 1/\mu} = e^{-2\rho},$$

where $1/\mu$ is the transmission time of a packet. Hence, the mean time before a succesful transmission is simply

$$\mathrm{E}[W] = \frac{1-p}{p} \cdot \Delta = \left(e^{2\lambda/\mu} - 1\right) \cdot \Delta,$$

where Δ denotes the mean of the backoff time which we for simplicity assume to be i.i.d. random variable. According to (11), for a small transmission range d we have

$$\lambda = q(\mathbf{r}, d) \approx 2d \cdot \Phi(\mathbf{r}).$$

Combining the above yields

$$W(\mathbf{r}) \approx \left(e^{4d\Phi(\mathbf{r})/\mu} - 1\right) \cdot \Delta,$$

which corresponds to mean spatial velocity of

$$\nu(\mathbf{r}) = \frac{d}{W + 1/\mu} = \frac{d \cdot \mu}{(e^{4d\Phi(\mathbf{r})/\mu} - 1) \cdot \Delta + 1}.$$

Consequently, the packet density at $\mathbf{r}$ is given by

$$n(\mathbf{r}) = \frac{\Phi(\mathbf{r})}{\nu(\mathbf{r})} = \frac{\left(e^{4d\Phi(\mathbf{r})/\mu} - 1\right) \cdot \Delta + 1}{d \cdot \mu} \cdot \Phi(\mathbf{r}). \tag{24}$$

Note that we deliberately neglect the retransmissions of other packets here and assume that the "background" traffic is still Poissonian.

6.1.4 Aloha with perfect transmission probability

In slotted Aloha with n nodes sharing a same channel the optimal transmission probability is $1/n$. With this a given node makes a succesful transmission in one time slot with probability of

$$p = \frac{1}{n} \cdot \left(1 - \frac{1}{n}\right)^{n-1} = \frac{(n-1)^{n-1}}{n^n}.$$

Consequently, the mean number of slots needed for a succesful transmission by a given node is

$$\mathrm{E}[X] = \frac{1}{p} = \frac{n^n}{(n-1)^{n-1}},$$

which, assuming each network node has at most one packet, corresponds to average packet velocity of

$$\nu(\mathbf{r}) = \frac{(n-1)^{n-1} \cdot d \cdot \mu}{n^n},$$

where $n \approx \pi d^2 \cdot n(\mathbf{r}) + 1$, d the transmission range and $1/\mu$ the time duration of the slot. Substituting the above into (12) gives,

$$\Phi(\mathbf{r}) = n(\mathbf{r}) \cdot \nu(\mathbf{r}) = \left(\frac{n-1}{n}\right)^n \cdot \frac{\mu}{\pi d}. \tag{25}$$

Note that for large n we have $(1-1/n)^n \to 1/e$, which, when substituted into (25), gives us the saturated flux

$$\Phi(\mathbf{r}) = \frac{\mu}{e \cdot \pi d}.$$

Hence, in this case the flux $\Phi(\mathbf{r})$ must be in range $[0, \mu/(e \cdot \pi \cdot d)]$ and the node density $n(\mathbf{r})$ is given by

$$n(\mathbf{r}) = \frac{g^{-1}(\Phi(\mathbf{r}) \cdot \pi \cdot d/\mu) - 1}{\pi\, d^2}, \tag{26}$$

where $g^{-1}(x)$ is the inverse function of $g(n) = \left(\frac{n-1}{n}\right)^n$.

6.1.5 *Approximation by Poisson arrivals and FIFO*

Our last approximation for estimating the traffic load and mean end-to-end delay takes a slightly different approach than the previous approximations. Let $\overline{N}$ denote the mean number of customers in the server. According to the PASTA property of Poisson arrivals, the mean waiting time in a FIFO-queue[1] is given by

$$\mathrm{E}[W] = 1/\mu \cdot \overline{N}.$$

With these in mind we suggest estimating the mean waiting time in a dense wireless multihop network by

$$W^*(\mathbf{r}) = 1/\mu \cdot n(\mathbf{r}) \cdot \pi d^2.$$

The total time until the end of (succesful) transmission is $W^*(\mathbf{r}) + 1/\mu$, which yields average velocity of

$$\nu(\mathbf{r}) = \frac{\text{distance}}{\text{time}} = \frac{d \cdot \mu}{\pi d^2 \cdot n(\mathbf{r}) + 1}.$$

Substituting that into (12) yields

$$\Phi(\mathbf{r}) = n(\mathbf{r}) \cdot \frac{d \cdot \mu}{\pi d^2 \cdot n(\mathbf{r}) + 1},$$

from what we obtain an expression for packet density (cf. (21)),

$$n(\mathbf{r}) = \frac{\Phi(\mathbf{r})}{(\mu - \pi d \cdot \Phi(\mathbf{r})) \cdot d}. \tag{27}$$

6.2 Comparison of end-to-end delays

Consider a unit disk with transmission range of $d = 0.2$ and uniformly distributed network nodes. The resulting scalar packet flux can be easily determined and we can calculate the resulting packet density $n(\mathbf{r})$ assuming M/M/1-, M/D/1- or simplified Aloha approximation. Hence, using (18) we also obtain the resulting mean end-to-end delay which is illustrated in Figure 6. The rapidly increasing dashed curves correspond to simplified Aloha model where as the higher solid line correspond to M/M/1-approximation and the lower solid line to M/D/1-approximation. With very little traffic the mean end-to-end delay is approximately 4.527 according to all the approximations, which corresponds to the mean distance between two node pairs divided by 0.2.

[1] holds also for, e.g., random service order.

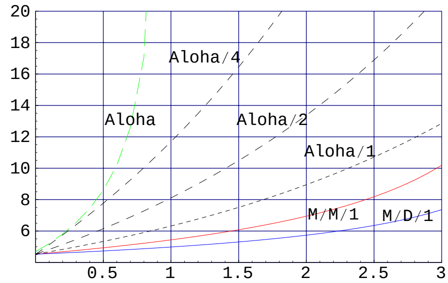


Figure 6: Mean end-to-end delay as a function of traffic load Λ in unit disk with transmission range $d = 0.2$ and uniformly distributed network nodes.

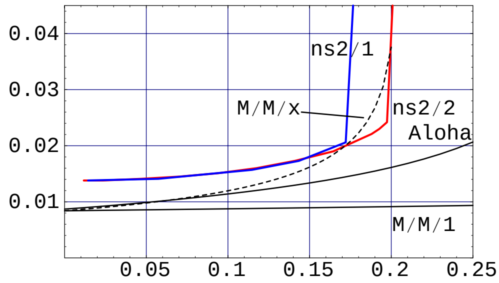


Figure 7: End-to-end delays in an wireless multihop network according to the analytical estimates and ns2-simulatations.

6.3 Queueing delays in 802.11 network

Our next numerical validation concerns the queueing delays. We have used a similar scenario as in Section 4.1 and have measured the mean end-to-end delay in the network with different levels of offered load. The simulated results (ns2/1 and ns2/2) as well the analytical estimates are illustrated in Figure 7. Case ns2/1 corresponds to pure CBR sources while ns2/2 corresponds to the exponential on/off sources which generates traffic with a constant bit rate while in on state. The analytical estimates are as follows. M/M/1 model is according to (21), M/M/x is according to (22) with $\rho_{\max} = 0.145$ (fitted to an experimental curve), and Aloha model corresponds to (26).

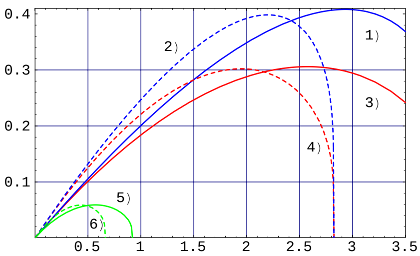
From Figure 7 it can be seen that pure M/M/1 model is highly optimistic in this situation as expected. With a proper choice of $\rho_{\max}$ (or $\Phi_{\max}$) the model can be adjusted to fit the actual situation reasonably well. Aloha model is somewhere inbetween these two. Note that the error at very small traffic load is due to the fact that the mean number of hops is rather small (about 2.5) in this case. This means that the truncation error we make at estimating the number of hops is considerably large (e.g., if the route length is 2.5, it requires 3 transmissions, not 2.5).

6.4 Delay-throughput analysis

Our next objective is to study the relationship between the transmission range d and the optimal traffic load $\rho^{(\mathrm{opt})}$ using elementary models for the queueing delays introduced in Section 6.1. Consider the following 6 scenarios for a dense multihop network in unit disk:

	model	nodes		model	nodes
1)	M/D/1	uniform	4)	M/M/1	RWP
2)	M/D/1	RWP	5)	Aloha	uniform
3)	M/M/1	uniform	6)	Aloha	RWP

RWP means that the nodes are assumed to be distributed accord-



scenario		max. power, γ
1)	M/D/1 + uniform nodes	0.408
2)	M/D/1 + RWP nodes	0.398
3)	M/M/1 + uniform nodes	0.306
4)	M/M/1 + RWP nodes	0.302
5)	Aloha + uniform nodes	0.059
6)	Aloha + RWP nodes	0.059

Figure 8: Graph illustrates the power as a function of offered load $\rho^{(\mathrm{tot})}$ for transmission range of $d = 0.2$. The maximum achievable power, which is independent of d, is shown in the table.

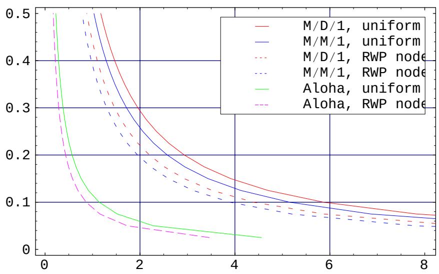


Figure 9: Optimal transmission range d as a function of traffic load $\rho^{(\mathrm{tot})}$ for different models.

ing to basic RWP, while uniform corresponds to uniform node distribution. In Figure 8 the resulting power γ is depicted for $d = 0.2$ as a function of traffic load ρ. From the figure it can be seen that there is clearly an explicit maximum for each case. Moreover, the maximum achievable power seems to depend more on the congestion model (M/M/1 or M/D/1) than the distribution of the network nodes (uniform or according to RWP model).

By evaluating each scenario numerically for different values of d it turns out that the maximum achievable power $\gamma_{\max}$ is independent of the transmission range d. In other words, the optimal point maximising the power (the ratio between throughput, $\lambda^{(\mathrm{tot})} \cdot B$, and the mean delay $\mathrm{E}[T]$) is constant for any value of d. Hence, for a given traffic topology $\mathcal{T}$ defining relative traffic intensities the maximum achievable power is a certain constant and we have

$$\mathrm{E}[T] = \gamma(\mathcal{T}) \cdot \rho^{(\mathrm{opt})} = \frac{\gamma(\mathcal{T})}{\mu} \cdot \lambda^{(\mathrm{opt})}.$$

In Figure 9 the optimal transmission range d is depicted as a function of total traffic load $\rho^{(\mathrm{tot})}$. As the traffic load increases the transmission range d must be decreased in order to satisfy the increased capacity requirements, which explains the shape of the curves.

Based on the rather surprising discovery made in the previous example it appears that the achievable maximum power in a dense wireless multihop network is independent of the transmission range given a high enough node density and fixed relative traffic intensities between the nodes (or locations). If this property holds for a given MAC protocol and traffic scenario then there is a linear relationship between the obtainable capacity and mean delay,

$$C = \gamma^* \cdot \mathrm{E}[T],$$

i.e., doubling the capacity (e.g. by adjusting the transmission power appropriately) means you will have to tolerate a two times longer mean transmission delay.

7. CONCLUSIONS

In this paper we have considered a dense multihop network with an assumption that the routes can be modelled as straight line segments. The movement of a packet is modelled by the so-called line segment traversing process. Using the model we have first analysed the relayed traffic load in different parts of the network, which has been validated through ns2 simulations. One special property of our model is the spatial velocity component, which is used to capture the queueing delays due to congestion in the network. By a proper choice of spatial velocity, the model can be adapted to any given MAC protocol. Additionally, we have given expressions for the mean one-way delay in the network, which is itself an important performance measure of the network, and for the ratio between throughput and the mean end-to-end delay (power). The results have been illustrated and validated by numerical examples and ns2-simulations.

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