Helsinki University of Technology
Signal Processing Laboratory

S-38.411 Signal Processing in Telecommunications I

Spring 2000

Lecture 8: Nonlinear receivers 1: DFE equalizers

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Timetable

L1 Introduction; models for channels and communication systems
L2 Channel capacity
L3 Transmit and receive filters for bandlimited AWGN channels
L4 Optimal linear equalizers for linear channels 1
L5 Optimal linear equalizers for linear channels 2
L6 Adaptive equalizers 1
L7 Adaptive equalizers 2
L8 Nonlinear receivers 1: DFE equalizers
L9 Nonlinear receivers 2: Viterbi algorithm
L10 GL1: DSP for Fixed Networks / *Matti Lehtimäki, Nokia Networks*
L11 GL2: DSP for Digital Subscriber Lines / *Janne Väänänen, Tellabs*
L12 GL3: DSP for CDMA Mobile Systems / *Kari Kalliojärvi, NRC*
L13 Course review, questions, feedback
E 24.5. (Wed) 9-12 S4 **Exam**

Contents of Lecture 8

I. Basic idea of decision-feedback equalization

Basic idea of DFE

- In the previous lectures, *linear* equalizers and their adaptive implementations have been studied
- Linear MMSE equalizer in stochastic gradient (=LMS) adaptive implementation is simple, efficient and robust
- Problem: performance is not always good enough
  - noise enhancement in channels with zeroes
  - long impulse responses are a problem
- Goal of this lecture:

  Improve the linear equalizer by simple nonlinear modifications

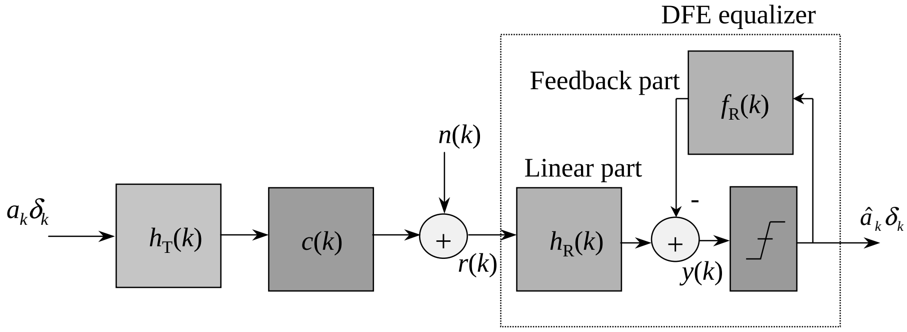
Basic idea of DFE...

- Linear equalizer processes input signal samples $r(k)$ only
- Noise always limits the performance
- Noise enhancement problem (particularly with ZF equalizer)
- Basic problem in linear filtering: desired signal and noise processed together
- New approach:

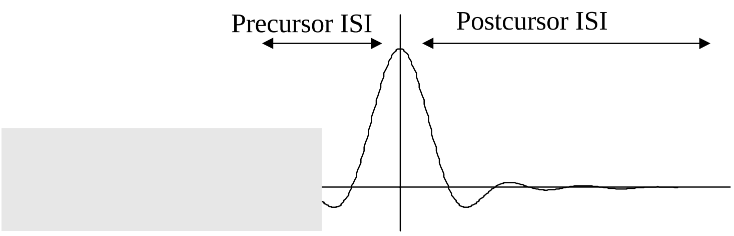
  Utilize previous symbol decisions $\hat{a}_k$ to cancel ISI!

Basic idea of DFE...

◆ Basic structure of a DFE equalizer (symbol-rate discrete-time model):



Basic idea of DFE...



◆ Channel impulse response:
  – at any decision instant, there is ISI contribution from some 'future' symbols (precursor ISI) and past symbols (postcursor ISI)
  – in causal systems, postcursor ISI can be moved by subtracting the weighted symbol value (*decision feedback)*

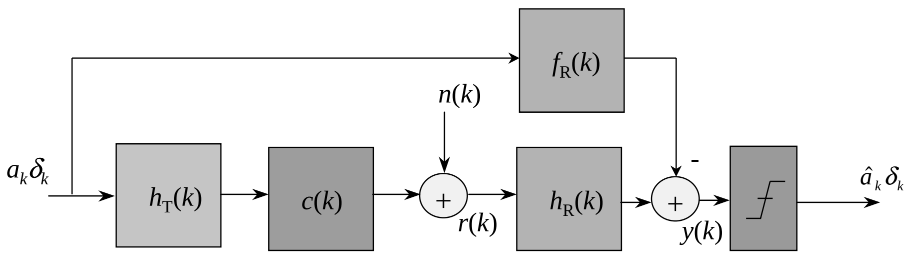
II. Design of DFE filters

Design of DFE filters...

- Both $h_R(k)$ and $f_R(k)$ are *linear* filters
- Complete filtering system is *nonlinear*, because nonlinear operation (symbol decision) is in the feedback loop
- For the design of DF Equalizer, both linear part (feed-forward) filter and feedback filter need to be determined
- For easier analysis and design, the system is *linearized* by assuming all decisions correct $(\hat{a}_k = a_k)$

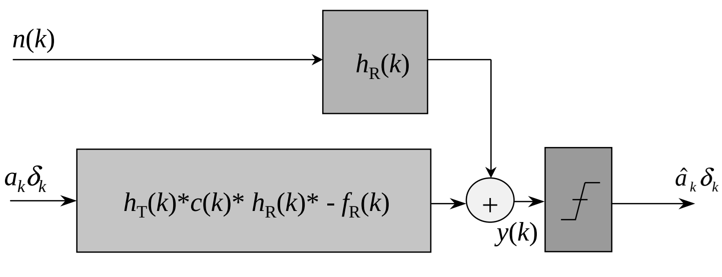
Design of DFE filters...

- Equivalent block diagram for *linearized* system:



Design of DFE filters...

- Simplified version, with separated signal and noise paths:



Design of DFE filters...

- Zero-forcing DFE:

$$h_T(k) * c(k) * h_R(k) - f_R(k) = \delta_k$$

$$\Leftrightarrow H_T(z)C(z)H_R(z) - F_R(z) = 1$$

- Several possible solutions for linear $h_R(k)$ and feedback filters $f_R(k)$ in the general case (infinite-length filters)
- Noise gain depends on choice of linear filter $h_R(k)$

Design of DFE filters...

- MMSE-DFE: minimize total mean square error $\mathrm{E}\left[e^2(k)\right]$

$$\begin{aligned} e(k) &= y(k) - a_k \\ &= \left[h_T(k) * c(k) * h_R(k) - f_R(k) - \delta_k\right] * a_k + h_R(k) * n(k) \end{aligned}$$

- Assume independent signal and noise:

$$\begin{aligned} \mathrm{E}\left[e^2(k)\right] = {} & \mathrm{E}\left[\left(\left[h_T(k) * c(k) * h_R(k) - f_R(k) - \delta_k\right] * a_k\right)^2\right] \\ & + \mathrm{E}\left[\left(h_R(k) * n(k)\right)^2\right] \end{aligned}$$

Design of DFE filters...

- Express MSE in frequency domain:

$$\mathrm{E}\left[e^2(k)\right] = \int \left| H_\mathrm{T}(f)C(f)H_\mathrm{R}(f) - F_\mathrm{R}(f) - 1 \right|^2 S_x(f)df$$

$$+ \int \left| H_\mathrm{R}(f) \right|^2 S_n(f)df$$

- MSE = MIN when:

$$\int \left| H_\mathrm{R}(f) \right|^2 S_n(f)df = \mathrm{MIN}$$

$$\text{with } H_\mathrm{T}(f)C(f)H_\mathrm{R}(f) - F_\mathrm{R}(f) = 1$$

Design of DFE filters...

Procedure for MMSE-DFE filter design:

1) Design linear filter $H_\mathrm{R}(f)$ so that noise is minimized
2) Design feedback $F_\mathrm{R}(f)$ filter so that ISI = zero

- In the ideal case (infinite-length feedback filter), all ISI can be completely eliminated!
- In practice, only *postcursor ISI* from a finite number of previous decisions can be eliminated
- Precursor ISI can be reduce by linear (precursor) filter and adding delay in the system

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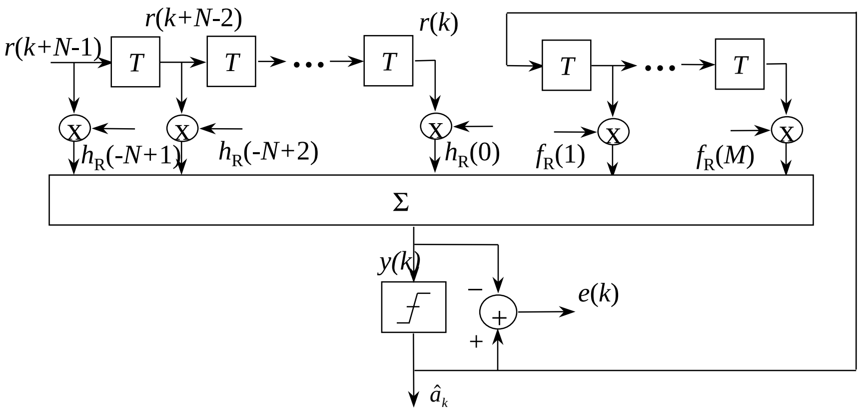
III. Adaptive DFE filters

Adaptive DFE filters

- Practical DFE filters ($h_R(k)$ and $f_R(k)$) are FIR filters
- Both linear and feedback filters are adjusted adaptively
- The adaptation can be done jointly for both $h_R(k)$ and $f_R(k)$ as if for a single FIR filter

Adaptive DFE filters...

- Signal flow diagram of adaptive DFE filter with (N+M) taps:



Adaptive DFE filters...

- Adaptive DFE algorithm:
  Define *augmented* signal and coefficient vectors

$$\mathbf{h}_{\mathrm{R}}^{+} = \left[h_R(-N+1) \;\; \cdots \;\; h_R(0) \;\; -f_R(1) \;\; \cdots \;\; -f_R(M)\right]^{\mathrm{T}}$$

$$\mathbf{r}^{+}(k) = \left[r(k+N-1) \;\; \cdots \;\; r(k) \;\; a_{k-1} \;\; \cdots \;\; a_{k-M}\right]^{\mathrm{T}}$$

- DFE output signal: $y(k) = \mathbf{h}_{\mathrm{R}}^{+\mathrm{T}}(k)\mathbf{r}^{+}(k)$

- Error signal: $e(k) = a_k - y(k) = a_k - \mathbf{h}_{\mathrm{R}}^{+\mathrm{T}}(k)\mathbf{r}^{+}(k)$

Adaptive DFE filters...

- Stochastic gradient algorithm for DFE:

$$\mathbf{h}_\mathrm{R}^+(k+1) = \mathbf{h}_\mathrm{R}^+(k) - \frac{\beta}{2}\nabla_{\mathbf{h}_\mathrm{R}} e^2(k)$$
$$= \mathbf{h}_\mathrm{R}^+(k) + \beta e(k)\mathbf{r}^+(k)$$

- Formally and computationally simple like linear SG algorithm!
- Convergence properties similar to linear SG algorithm

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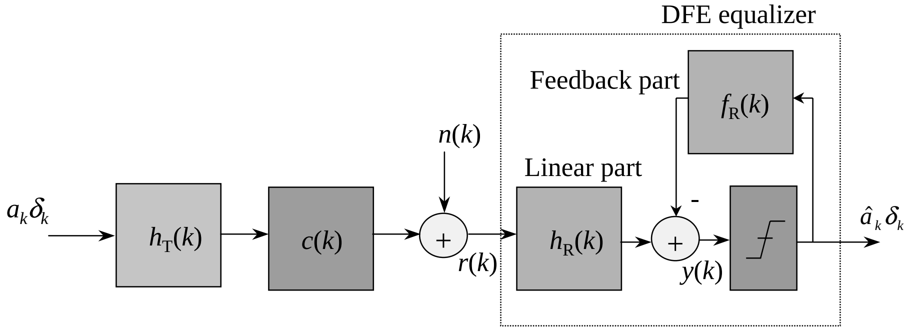
IV. Error propagation

Error propagation

- Linearized DFE model assumes all decisions *correct*
- What happens if they are not?



Error propagation...

- One decision error in DFE causes a *burst of new errors*
- The errors only stop after M (= order of feedback filter) *consecutive* correct decisions
- It can be shown (see Lee-Messerschmitt Appendix 10-A) that this happens after K symbols in the average, where

$$K = 2(2^M - 1)$$

- This gives average error probability

$$P_e = 2^M P_{e,0}$$

where $P_{e,0}$ is the error probability with no error propagation

Error propagation...

- Error propagation is the major problem in DFE
- It can be kept in control by keeping the error probability low (with other system choices) and keeping the feedback filter short enough
- Note! The error probability after DFE *cannot* be improved by error-correcting coding! (Why?)
- Alternative for DFE: *Tomlinson-Harashima precoding*
  - move feedback part in the transmitter
  - more in the DSL guest lecture!

Summary

Today we discussed:

Nonlinear receivers 1: DFE equalizers

I. Basic idea of decision-feedback equalization

II. Design of DFE filters

III. Adaptive DFE

IV. Error propagation

Next week: Viterbi algorithm