S-38.411 Signal Processing in Telecommunications I
Exercise #2: Matched filter

February 25, 2000

Figure 1 below shows the block diagram of a communication system to be studied in this exercise.

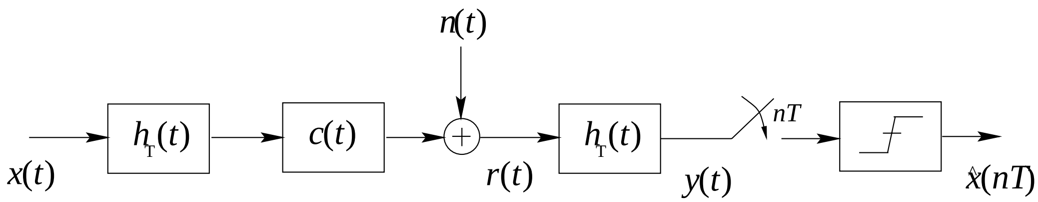

Figure 1: Block diagram over communications system

- $x(t)$ is the input signal (the information we want to recover).
- $h_T(t)$ and h_R are the transmit and receive filters
- $c(t)$ is the channel impulse response
- $n(t)$ is additive noise (either white or colored noise)

The topics in Lecture 2-3 were to how to design the receive filter $h_R(t)$ given knowledge about $h_T(t)$, $c(t)$, $n(t)$. The cases of of AWGN channel, and linear channel with colored noise were treated. We will in this exercise look at two different solutions

1. The matched filter (MF), which maximize the SNR.
2. The Nyquist filter, which combine the MF with the Nyquist criterion.

1. Generalized Matched Filter (GMF)

From Lecture 4 we have

$$SNR = \frac{g^2(0)}{\mathrm{E}\left[n_R(t)\right]} = \frac{\left|\int_{-\infty}^{\infty} H_T(f)C(f)H_R(f)e^{-j2\pi ft}\, df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} \left|H_R(f)S_{n_0}(f)\right|^2\, df}$$
$$\leq \frac{2}{N_0}\int_{-\infty}^{\infty}\left|\frac{H_T(f)C(f)}{\sqrt{S_{n_0}(f)}}\right|^2 df = SNR_{\mathrm{MAX}} \tag{1}$$

with equality only when

$$H_R(f) = k_0\frac{H_T^*(f)C^*(f)}{S_{n_0}(f)} \overset{\mathcal{F}^{-1}}{\rightarrow} h_R(t) = k_0 h_T(-t) \star c(-t) \star n_I(t) \tag{2}$$

Special cases:

i) AWGN channel ($S_{n_0}(f) = 1$) $\rightarrow H_R(f) = H_T^*(f)$

ii) Linear channel $\rightarrow H_R(f) = H_T^*(f)C^*(f)$

2. Matched filter with Nyquist

The matched filter does not consider the problem of inter-symbol interference (ISI). If we require zero ISI at the receiver, the combined response given by

$$G(f) = H_T(f)C(f)H_R(t) \tag{3}$$

has to be a Nyquist spectrum ($G_N(f)$). From Lecture 4, we finally have

$$H_T(f) = \frac{\sqrt{G_N(f)S_{n_0}(f)}}{C(f)}e^{-j\phi}$$
$$H_R(f) = \sqrt{\frac{G_N(f)}{S_{n_0}(f)}}e^{j\phi} \tag{4}$$

Special cases:

i) AWGN channel ($S_{n_0}(f) = 1$) ($G_N(f)$) real-valued and positive $\rightarrow H_R(f) = H_T(f) = \sqrt{G_N(f)}$

ii) Linear channel $\rightarrow H_T(f) = \frac{\sqrt{G_N(f)}}{C(f)}$

Exercise 1:

a) Consider an AWGN channel. The pulse shape $h_T(t)$ of a transmitter filter is given to the left in Figure 2 below. Calculate the frequency response $H_T(f)$, and design the receive filter $h_R(t)$ that maximize the SNR. Is the solution causal?

b) Now consider both the transmit filter $h_T(t)$ and the channel impulse response in Figure 2. Calculate the frequency response of the channel $C(f)$, and design the receive filter $h_R(t)$ that maximizes the SNR

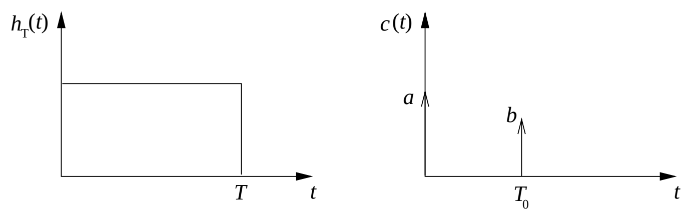

Figure 2: Transmit pulse and channel impulse response for Exercise 1.

Exercise 2:

The spectrum for a pulse satisfying the Nyquist criterion is shown in Figure 3 below. Assume colored noise with $S_{n_0}(f) = \frac{1}{1+|f|}$.

a) For the channel $c(t) = 0.5\delta(t) + 0.5\delta(t - T_0)$ calculate the spectra ($H_R(f)$ and $H_T(f)$) for the GMF. Any problem?

b) Repeat a) for the channel $c(t) = 0.5\delta(t) - 0.5\delta(t - T_0)$. Any problem?

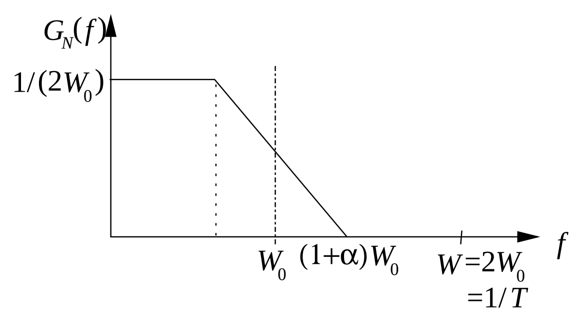

Figure 3: Nyquist spectrum for Exercise 2.

Homework

The homework is to be returned to the course box *at latest* March 10, 15:00. The course box can be found near the course information board on the second floor in the G wing. Each set of homework can give up to 1 point on the final exam. Remember to motivate each step in your solution. Write your name and student number on each page.

1. a) Design a Nyquist spectrum (as simple as possible) that uses the bandwidth $(\frac{-3W_0}{2}, \frac{3W_0}{2})$ excluding $\frac{W_0}{2} \leq |W| \leq \frac{3W_0}{4}$.

   b) Design corresponding root-Nyquist filters (both spectra and pulse waveforms).

   c) Given the channel response $C(f) = 2 + \cos 2\pi f T$, and the noise PSD $S_n(f) = \sin \pi f T$, design optimal GMF with Nyquist filters. Any problems?