S-38.411 Signal Processing in Telecommunications I
Exercise #1: Channel capacity

February 18, 2000

The capacity of a linear channel with colored noise is given by

$$C = \int_0^\infty \log_2 \left(1 + \frac{S_x(f)|C(f)|^2}{S_n(f)}\right) df = \int_{-\infty}^\infty \frac{1}{2} \log_2 \left(1 + \frac{S_x(f)|C(f)|^2}{S_n(f)}\right) df \tag{1}$$

To calculate the capacity for a given channel $C(f)$ and noise spectrum $S_n(f)$ requires knowledge of signal spectrum $S_x(f)$. In Lecture 2 it was shown that for a given transmit power P_x, the maximum capacity is obtained when

$$S_{x,\mathrm{opt}}(f) = L - \frac{S_n(f)}{|C(f)|^2} \tag{2}$$

where the L can be solved from the power constraint

$$P_x = \int_0^\infty S_x(f)\, df = \int_0^\infty \left(L - \frac{S_n(f)}{|C(f)|^2}\right) df \tag{3}$$

Note that the AWGN channel is a special case of the above (see Lecture 2).

Recipe for solving capacity problems:

1. Solve for L using (3)
2. Calculate $S_{x,\mathrm{opt}}(f)$ using (2)
3. Calculate $C_{\max}$ using (1)

Exercise 1:

Consider the constant gain channel of bandwidth W:

$$\begin{aligned} S_n(f) &= \frac{P_n}{W}, \ 0 \le f \le W \\ C(f) &= A, \ 0 \le f \le W \end{aligned} \tag{4}$$

The channel above corresponds to the AWGN channel when $A = 1$. For a given transmission power P_x, calculate the maximum capacity $C_{\max}$ and $S_{x,\mathrm{opt}}(f)$.

Exercise 2:

This exercise considers a channel with a zero-gain in the low frequency band, and a constant gain in the high frequency band.

$$\begin{aligned} S_n(f) &= \frac{P_n}{W}, \ 0 \le f \le W \\ C(f) &= \begin{cases} 0, & 0 \le f < W_0 \\ A, & W_0 \le f \le W \end{cases} \end{aligned} \tag{5}$$

For a given transmission power P_x, calculate for the maximum capacity $C_{\max}$ and $S_{x,\mathrm{opt}}(f)$. Let $W_0 \to 0$. Is the result consistent with that of the AWGN channel?

Exercise 3: (from Lecture 2)

Consider the two-band channel of bandwidth W:

$$\begin{aligned} S_n(f) &= \frac{P_n}{W}, \ 0 \le f \le W \\ C(f) &= \begin{cases} A, & 0 \le f < W_0 \\ B, & W_0 \le f \le W \end{cases} \end{aligned} \tag{6}$$

Calculate for the maximum capacity $C_{\max}$ and $S_{x,\mathrm{opt}}(f)$.

Homework

The homework is to be returned to the course box *at latest* March 3, 15:00. The course box can be found near the course information board on the second floor in the G wing. Each set of homework can give up to 1 point on the final exam. Remember to motivate each step in your solution. Write your name and student number on each page.

1. Consider the channel in the figure below. Assume the transmitted power fixed to P_x.

   a) Calculate the maximum capacity $C_{\max}$ and $S_{x,\mathrm{opt}}(f)$.

   b) Compare the capacity obtained here with that of an AWGN channel ($|C(f)| = 1$ for $0 \leq f \leq W$).