1. This problem compares simple add/drop multiplexer architectures.
(a) First consider the fiber Bragg grating based add/drop element shown in Fig. 1. Suppose a $5 \%$ tap is used to couple the added signal into the output, and the grating induces a loss of 0.5 dB for the transmitted signals and no loss for the reflected signal. Assume that the circulator has a loss of 1 dB per pass. Carefully compute the loss seen by a channel that is dropped, a channel that is added, and a channel that is passed through the device. Suppose the input power per channel is $\mathbf{1 5 ~ d B m}$. At what power should the add channel be transmitted so that the powers on all channels at the output are the same?
(b) Suppose you had to realize an add/drop multiplexer that drops and adds four wavelengths. One possible way to do this is to cascade four add/drop elements on the type shown in Fig. 1 in series. In this case, compute the best-case and worst-case loss seen by a channel that is dropped, a channel that is added, and a channel that is passed through the device.


Figure 1: Optical add/drop elements based on fiber Bragg gratings.
(c) Another way to realize a four-channel add/drop multiplexer is shown in Figure 2. Repeat the preceding exercise for this architecture. Assume that the losses are as shown in the figure. Which of the two would you prefer from a loss perspective?
(d) Assume that fiber gratings cost $\mathbf{\$ 5 0 0}$ each, circulators $\mathbf{\$ 3 0 0 0}$ each, filters $\mathbf{\$ 1 0 0 0}$ each, and splitters, combiners and couplers $\$ 100$ each. Which of the two preceding architectures would you prefer from a cost point of view?


Figure 2: A four-channel add/drop multiplexer architecture.

## Solution:

Fiber Bragg grating is a simple device which reflects one wavelength channel and let through all the other. The coupler is illustrated in Fig. 3.
$5 \%$ equals to $10 \log _{10} 100 / 5=13 d B$ loss,
while $95 \% \rightarrow 0.2 d B$.
Figure 3: A $5 \%$ tap coupler.
(a) Thus, one gets,

| drop | $1 \mathrm{~dB}+1 \mathrm{~dB}=2 \mathrm{~dB}$ |
| :--- | :--- |
| add | 13 dB |
| through | $1 \mathrm{~dB}+0.5 \mathrm{~dB}+0.2 \mathrm{~dB}=1.7 \mathrm{~dB}$ |

If input power is 15 dBm , the output power will be $15-1.7 \mathrm{dBm}=13.3 \mathrm{dBm}$. Thus the power of added signal should $13.3+13 \mathrm{dBm}=\underline{26.3 \mathrm{dBm}}$.
(b) By using the results from previous point:

| Case | Worst Case | Best case |
| :--- | :--- | ---: |
| drop | $3 \cdot 1.7 \mathrm{~dB}+2 \mathrm{~dB}=7.1 \mathrm{~dB}$ | 2 dB |
| add | $13 \mathrm{~dB}+3 \cdot 1.7 \mathrm{~dB}=18.1 \mathrm{~dB}$ | 13 dB |
| through | $4 \cdot 1.7 \mathrm{~dB}=6.8 \mathrm{~dB}$ | 6.8 dB |

(c) The coupler has $10 \%$ combining ratio, and thus $10 \% \rightarrow 10 \mathrm{~dB}$ and $90 \% \rightarrow 0.5 \mathrm{~dB}$.

|  | Worst case |  | Best case |
| :--- | :--- | ---: | ---: |
| drop | $1 \mathrm{~dB}+6 \cdot 0.5 \mathrm{~dB}+1 \mathrm{~dB}+6 \mathrm{~dB}+1 \mathrm{~dB}$ | $=12 \mathrm{~dB}$ | 9 dB |
| add | $6 \mathrm{~dB}+10 \mathrm{~dB}$ | $=16 \mathrm{~dB}$ | 16 dB |
| through | $1 \mathrm{~dB}+4 \cdot 0.5 \mathrm{~dB}$ | $=3 \mathrm{~dB}$ | 3 dB |

This configuration seems better.
(d) i. $4 \cdot(500+3000+100)=\$ 14.4 \mathrm{k}$
ii. $4 \cdot 500+3000+4000+300=\$ 9.3 \mathrm{k}$
2. Construct a $16 \times 16$ broadcast star interconnecting 32 directional couplers along the lines given in slide 30 of lecture 8 .

## Solution:



Figure 4: $16 \times 16$ broadcast star. Each input port is directed to all output ports.

The solution is illustrated in Fig. 4. Note that there are also other equivalent constructions yielding the same result.
3. a) Consider the (static) $4 \times 4$ wavelength router with 4 wavelengths depicted in slide 31 of lecture 8 . There are 16 fibers between the input and output stages, connected in a way that prevents identical wavelengths from different input ports from being combined on the same output port, thus avoiding interference among the different channels. Formulate the routing rule used, i.e. tell how the output port number $i$ is determined from the input port number $\boldsymbol{j}$ and wavelength index $k$.
b) Generalize the rule to a $N \times N$ wavelength router with $N$ wavelengths.

## Solution:

So $j$ is the input port and $k$ is the wavelength. The output port is $i$.
a) Clearly,

$$
i=1+(j+k-2 \bmod 4)
$$

$$
\text { (or: } i-1=(j-1)+(k-1) \bmod 4) .
$$

b) Similarly,

$$
i=1+(j+k-2 \bmod N)
$$

(or: $i-1=(j-1)+(k-1) \bmod N$ ).


Figure 5: Static $4 \times 4$ wavelength router.
4. Consider the (dynamic) $4 \times 4$ wavelength selective cross-connect with 4 wavelengths depicted in slide 37 of lecture 8 . As you see, it contains 4 permutation switches, one for each wavelength. Determine the connection states of these four permutation switches that together correspond to the permanent state of the $4 \times 4$ wavelength router depicted in slide 31 of lecture 8. (Thus, for example, the wavelength $\lambda_{2}$ appearing in the input port 4 should be routed to the output port 1.)

## Solution:

Permutation switch is an element which directs each input to exactly one output so that no two inputs are directed to to the same output. Thus, for example the highest permutation switch handles the wavelength channel $\lambda_{1}$ etc. and we have the following connection states:

| Switch 1 |  |
| :---: | :---: |
| input | output |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |


| Switch 2 |  |
| :---: | :---: |
| input | output |
| 1 | 2 |
| 2 | 3 |
| 3 | 4 |
| 4 | 1 |

Switch 3

| input | output |
| :---: | :---: |
| 1 | 3 |
| 2 | 4 |
| 3 | 1 |
| 4 | 2 |

Switch 4

| input | output |
| :---: | :---: |
| 1 | 4 |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |

5. Realize the full connectivity between 3 end systems by interconnecting 3 wavelength add-drop multiplexers (see slide 45 of lecture 8) into a unidirectional ring and using wavelength routing to provide the optical connections needed. Solve the routing and channel assignment problem. How many wavelengths are needed (at minimum)? How many optical transceivers are needed in each NAS? Show the states of all add-drop switches in WADM1.

## Solution:



Node 1
Figure 6: Left figure illustrates the unidirectional ring. The graph on the right shows the corresponding relations between different connections, i.e. which connections must be assigned a different wavelength in order to avoid wavelength conflicts.

The routing in case of unidirectional (single fibre) ring is unambiguous and depicted in the figure. Hence, only the wavelength channel assignment (WA) needs to be done. In single fibre case the wavelength assignment is essentially the graph node coloring problem, which is known to be NP-hard problem. The graph corresponding to the given routing is shown on the right. Each node represents one connection and those connections which share the same fibre somewhere along the route are set as neighbour, i.e. they must be assigned unique channels.

As the graph is planar, the four color theorem ${ }^{1}$ quarantees that 4 wavelengths is enough. Also the graph consists of many cliques of size 3 and hence at least 3 wavelengths are needed. In this case a 3-coloring exists. Namely, e.g.

| connection | $1 \rightarrow 2$ | $1 \rightarrow 3$ | $2 \rightarrow 1$ | $2 \rightarrow 3$ | $3 \rightarrow 1$ | $3 \rightarrow 2$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| channel | 1 | 2 | 1 | 3 | 2 | 3 |

satisfies the requirements. The WADM1 is depicted in Fig. 7. It can be noted from the figure that at least 2 transceivers are needed in each node.


Figure 7 : WADM of node 1.

[^0]1. Imagine that you are a network planner for a major carrier that is interested in deploying a $20 \mathrm{~Gb} / \mathrm{s}$ link. You must choose between the following options:
(a) An SDM approach using eight fiber-pairs, each operating at $2.5 \mathrm{~Gb} / \mathrm{s}$.
(b) A TDM approach using two $10 \mathrm{~Gb} / \mathrm{s}$ transmission systems over two fiber pairs.
(c) A WDM approach using eight wavelengths over a fiber-pair.

Do an economic analysis to determine the costs of installing each type of system for a link length (a) 1200 km , (b) 240 km , and (c) 40 km . Use spreadsheet or a computer program so that you can vary the parameters and see how they affect your system choice.
Assume the following costs:

| Equipment | Cost $(\$)$ |
| :--- | :---: |
| $2.5 \mathrm{~Gb} / \mathrm{s}$ terminal | $\mathbf{1 0 0 . 0 0 0}$ |
| $10 \mathrm{~Gb} / \mathrm{s}$ terminal | $\mathbf{3 0 0 . 0 0 0}$ |
| 8-WDM terminal equipment | 100.000 |
| Optical amplifier pair | $\mathbf{1 0 0 . 0 0 0}$ |
| $2.5 \mathrm{~Gb} / \mathrm{s}$ regenerator pair | $\mathbf{1 0 0 . 0 0 0}$ |
| $10 \mathrm{~Gb} / \mathrm{s}$ regenerator pair | $\mathbf{1 5 0 . 0 0 0}$ |

The WDM terminal equipment includes only the multiplexing and demultiplexing equipment and any amplifiers needed, but does not include the $2.5 \mathbf{G b} /$ s terminal equipment that must be paid for seperately.
Assume that the chromatic dispersion limit is 600 km at $2.5 \mathrm{~Gb} / \mathrm{s}$ and 120 km at $10 \mathrm{~Gb} / \mathrm{s}$, and that the PMD limit at $10 \mathrm{~Gb} / \mathrm{s}$ is 600 km . Regeneration is required beyond these limits. An optical amplifier is required every 120 km , unless the link is terminated at that point, and the same type of amplifier can be used in all the systems. Instead of an amplifier, you can optionally use a regenerator, but one is required every 80 km . Assume that standard single-mode fibers are already installed and available for free.
What do you conclude from your study? How would your conclusions change if the cost of the $10 \mathrm{~Gb} / \mathrm{s}$ terminal equipment drops $25 \%$, the $2.5 \mathrm{~Gb} / \mathrm{s} 40 \%$, and the WDM terminal equipment by $25 \%$ ?

## Solution:

The required equipment is shown in the following table.

|  | 40 km |  | 240 km |
| :--- | ---: | ---: | ---: |
| SDM, $8 \times 2.5 \mathrm{~Gb} / \mathrm{s}$ | $16 T$ | $16 T+8 A$ | $16 T+72 A+16 R$ |
| TDM, $2 \times 10 \mathrm{~Gb} / \mathrm{s}$ | $4 T_{10}$ | $4 T_{10}+2 R_{10}+2 A$ | $4 T_{10}+28 R_{10}$ |
| WDM, $8 \times 2.5 \mathrm{~Gb} / \mathrm{s}$ | $16 T+2 W$ | $16 T+2 W+A$ | $16 T+6 W+10 A+8 R$ |

The prices before and after drop are,

| Equipment | Old (k\$) | New (k\$) | drop (k\$) |
| :--- | ---: | ---: | ---: |
| $2.5 \mathrm{~Gb} / \mathrm{s}$ terminal | 100 | 60 | 40 |
| 10 Gb/s terminal | 300 | 225 | 75 |
| 8-WDM terminal equipment | 100 | 75 | 25 |



Figure 8: Possible locations of amplifiers and generators in 1200km long link.

With the old prices we get,

|  | 40 km |  | 240 km |
| :--- | ---: | ---: | ---: |
| SDM, $8 \times 2.5 \mathrm{~Gb} / \mathrm{s}$ | 1.6 | $1.6+0.8=2.4$ | $1.6+7.2+1.6=10.4$ |
| $\mathrm{TDM}, 2 \times 10 \mathrm{~Gb} / \mathrm{s}$ | 1.2 | $1.2+0.3+0.2=1.7$ | $1.2+4.2=5.4$ |
| WDM, $8 \times 2.5 \mathrm{~Gb} / \mathrm{s}$ | $1.6+0.2=1.8$ | $1.6+0.2+0.1=1.9$ | $1.6+0.6+1.0+0.8=4.0$ |

and after the drop,

|  | 40 km | 240 km | 1200 km |
| :--- | ---: | ---: | ---: |
| $\mathrm{SDM}, 8 \times 2.5 \mathrm{~Gb} / \mathrm{s}$ | $1.6-0.64=0.96$ | $2.4-0.64=1.76$ | $10.4-0.64=9.76$ |
| $\mathrm{TDM}, 2 \times 10 \mathrm{~Gb} / \mathrm{s}$ | $1.2-0.3=0.90$ | $1.7-0.3=1.40$ | $5.4-0.3=5.10$ |
| WDM, $8 \times 2.5 \mathrm{~Gb} / \mathrm{s}$ | $1.6-0.79=1.43$ | $1.9-0.79=1.11$ | $4.0-1.09=2.91$ |

2. a) Consider a static broadcast star network consisting of 5 NAS's and using TDM/TWDMA in FT-TR mode. Find a channel allocation schedule that realizes full point-topoint logical connectivity between the 5 stations using 5 wavelengths.
b) Assume then that the broadcast star operates in TT-FR mode. Find again a channel allocation schedule that realizes full point-to-point logical connectivity between the 5 stations using 5 wavelengths.

## Solution:

- The transmit channel was fixed, so we assign channel $i$ for node $i$ first. Then time slots must be assigned so that each node is receiving only in one channel in each time slot:

| channel | transmit | receive - slot |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 |  |
| $\lambda_{1}$ | 1 | $1 \rightarrow 2$ | $1 \rightarrow 3$ | $1 \rightarrow 4$ | $1 \rightarrow 5$ |  |
| $\lambda_{2}$ | 2 | $2 \rightarrow 3$ | $2 \rightarrow 4$ | $2 \rightarrow 5$ | $2 \rightarrow 1$ |  |
| $\lambda_{3}$ | 3 | $3 \rightarrow 4$ | $3 \rightarrow 5$ | $3 \rightarrow 1$ | $3 \rightarrow 2$ |  |
| $\lambda_{4}$ | 4 | $4 \rightarrow 5$ | $4 \rightarrow 1$ | $4 \rightarrow 2$ | $4 \rightarrow 3$ |  |
| $\lambda_{5}$ | 5 | $5 \rightarrow 1$ | $5 \rightarrow 2$ | $5 \rightarrow 3$ | $5 \rightarrow 4$ |  |

- In this case the receiving channels was fixed. Hence for example the following time slot assignment works:

| channel | receive | transmit - slot |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 |  |
| $\lambda_{1}$ | 1 | $2 \rightarrow 1$ | $3 \rightarrow 1$ | $4 \rightarrow 1$ | $5 \rightarrow 1$ |  |
| $\lambda_{2}$ | 2 | $3 \rightarrow 2$ | $4 \rightarrow 2$ | $5 \rightarrow 2$ | $1 \rightarrow 2$ |  |
| $\lambda_{3}$ | 3 | $4 \rightarrow 3$ | $5 \rightarrow 3$ | $1 \rightarrow 3$ | $2 \rightarrow 3$ |  |
| $\lambda_{4}$ | 4 | $5 \rightarrow 4$ | $1 \rightarrow 4$ | $2 \rightarrow 4$ | $3 \rightarrow 4$ |  |
| $\lambda_{5}$ | 5 | $1 \rightarrow 5$ | $2 \rightarrow 5$ | $3 \rightarrow 5$ | $4 \rightarrow 5$ |  |

3. Consider a static network connecting four stations with the following normalized traffic matrix:

$$
T=\left(\begin{array}{cccc}
0 & 1 & 1 & 1 \\
1 & 0 & 1 & 3 \\
2 & 2 & 0 & 1 \\
2 & 1 & 1 & 0
\end{array}\right)
$$

The network is realized as a TDM/T-WDMA system operating on a $4 \times 4$ broadcast star with $3 \lambda$-channels. Assume further that there is one transmitter and one receiver in each station.
a) Find an optimal channel allocation schedule for the system operating in FT-TR mode.
b) Find an optimal channel allocation schedule for the system operating in TT-FR mode.

## Solution:

First, the total traffic to and from each station is,

| station | transmit | receive |
| :--- | :---: | :---: |
| 1 | 3 | 5 |
| 2 | 5 | 4 |
| 3 | 5 | 3 |
| 4 | 4 | 5 |

Furthermore, total 3 wavelength channels are available: $\lambda_{1}, \lambda_{2}$ and $\lambda_{3}$.
a) FT-TR: fixed transmitter, tunable receiver

First we note that two transmitters must be operating in the same wavelength. If we neglect the receivers, it can be deduced that no better configuration is possible than one which assigns same wavelength for stations 1 and 4 , constituting period of 7 slots.

| channel | time slot |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\lambda_{1}$ | $1 \rightarrow 2$ | $1 \rightarrow 3$ | $1 \rightarrow 4$ | $4 \rightarrow 1$ | $4 \rightarrow 1$ | $4 \rightarrow 2$ | $4 \rightarrow 3$ |
| $\lambda_{2}$ | $2 \rightarrow 1$ | $2 \rightarrow 4$ | $2 \rightarrow 3$ | $2 \rightarrow 4$ | $2 \rightarrow 4$ |  |  |
| $\lambda_{3}$ | $3 \rightarrow 4$ | $3 \rightarrow 1$ | $3 \rightarrow 1$ | $3 \rightarrow 2$ | $3 \rightarrow 2$ |  |  |

Note that in any time slot each receiver is used at most once.
b) TT-FR: tunable transmitter, fixed receiver

Here the bottleneck is the receivers. Similarly it looks promising to try a configuration where the receivers of the stations 2 and 3 share the same wavelength. Allocating first the wavelength $\lambda_{1}$ to them gives (for example) the following allocation (which is optimal).

| channel | time slot |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\lambda_{1}$ | $1 \rightarrow 2$ | $3 \rightarrow 2$ | $3 \rightarrow 2$ | $4 \rightarrow 2$ | $1 \rightarrow 3$ | $2 \rightarrow 3$ | $4 \rightarrow 3$ |
| $\lambda_{2}$ | $2 \rightarrow 1$ | $4 \rightarrow 1$ | $4 \rightarrow 1$ | $3 \rightarrow 1$ | $3 \rightarrow 1$ |  |  |
| $\lambda_{3}$ | $3 \rightarrow 4$ | $1 \rightarrow 4$ | $2 \rightarrow 4$ | $2 \rightarrow 4$ | $2 \rightarrow 4$ |  |  |

Note that in any time slot each transmitter is used at most once.
4. a) Realize the full connectivity between 3 end systems by interconnecting $32 \times 2$ wavelength selective cross-connects (see slide 37 of lecture 8 ) into a unidirectional ring and using wavelength routing to provide the optical connections needed. Solve the routing and channel assignment problem. How many wavelengths are needed (at minimum)? How many optical transceivers are needed in each NAS? Show the connection states of all permutation switches in WSXC1.
b) Assume then that the 3 WSXC's are connected into a bidirectional ring. Make a similar study as above.

## Solution:



Figure 9: Left figure illustrates the unidirectional ring and the right figure bidrectional ring.
a) The optimal RWA is the same as in problem $5 / 5$, i.e. 3 wavelength channels are needed. The configuration of WSXC at node 1 is illustrated in Fig. 10. Two optical transceivers is sufficient in each node.
b) In this case the shortest path routing gives that there is only one connection in each link (to both directions). Assuming $4 \times 4$ WSXC and nonblocking NAS's with two access fiber pairs, only one wavelength channel is clearly sufficient. Also in this case two optical transceivers is required in each NAS.
5. a) Consider the mesh WRN consisting of 5 WSXC's and 5 elementary NAS's depicted in slide 22 of lecture 9 . Solve the routing and channel assignment problem that realizes the full logical/optical connectivity between the 5 stations using 4 wavelengths.
b) Assume then that the NAS's are nonblocking (as in slide 23 of lecture 9). Solve the same routing and channel assignment problem as above using only 2 wavelengths. Why is it impossible to solve the problem using only a single wavelength?


Figure 10: WSXC at node 1.

## Solution:

a) In the Fig. 11 two possible routings are illustrated. In figure on the left the "diagonal" connections travel anti-clockwise in the "border" of the network, while in the figure on the right the "diagonal" connections go through node 5 . In both figures only subset of connections is depicted for clarity. Note that only "diagonal" connections have several alternative shortest routes. For the rest the shortest route is always used.


Figure 11: Two possible routings.
The routings lead to the following wavelength assignment problems (graph node coloring). Immediately it can be noticed that at least 4 colors are needed as both graphs have several cliques ${ }^{2}$ of size 4 . Feasible 4 -colorings for both routings exist and they are depicted in Fig. 13. Thus four wavelength channels is sufficient in this case.
b) In this case NAS's are nonblocking, i.e. one does not need to consider the link between ONN and NAS. The routing where "diagonal" connections go through node 5 is clearly advantageous in this case as there are max. two connections routed to the same link (in same direction). As connections to/from NAS can be neglected, we are left with a simple wavelength assignment problem depicted in the Fig. 14. Hence, two wavelength channels is sufficient in this case.

## Lower bound for the number of wavelengths needed

As from each node $i, i=1, \ldots, 4$, total 4 connections emerge using 3 links, it is obvious that at least in some link there must be 2 connections and hence a single wavelength configuration is impossible.

[^1]

Figure 12: Wavelength assignment problems depicted as graph node coloring.


Figure 13: Feasible wavelength assignments for both routings with 4 wavelengths.


Figure 14: Wavelength assignment with nonblocking NAS's is simple.

1. Static networks: packet switching in the optical layer.
a) The simplest possible MAC protocol is random access (ALOHA). In the slotted version (slotted ALOHA), based on a slot-synchronized, single-channel shared broadcast medium, any station having a packet to transmit sends it in the next available slot. Assume that there are $N$ independent stations and each station transmits into each slot with probability $p$. Determine the probability $p_{s}(N)$ that a transmission is succesful. Let then $N \rightarrow \infty$ and $p \rightarrow 0$ in such a way that $N p \rightarrow \rho$ (where $\rho$ refers to the offered load in packets per slot). Determine the throughput $\theta=\rho p_{s}(\infty)$ in this limiting case and find its maximum as a function of $\rho$.
b) A bit more advanced MAC protocol is called tell-and-go. Now each station with a packet to transmit first communicates to all receivers the destination of the packet, and then sends the packet in the next time slot. All the collisions can be avoided using TDM-T/WDMA in FT-TR mode. Although conflicts cannot be avoided, the receiver can always choose one of the conflicting packets. Assume again that there are $N$ independent stations and each station transmits into each slot with probability $p$ (uniformly to the other stations). Determine the probability $p_{t}(N)$ that at least one station is transmitting to receiver $j$ in a timeslot. Let then $N \rightarrow \infty$ (keeping, however, $p$ constant). Determine the throughput per receiver, $\theta=p_{t}(\infty)$, in this limiting case and find its maximum as a function of the offered load per receiver, $\rho=p$.

## Solution:

a) Random access: (slotted ALOHA)

We have $N$ stations each transmitting in an arbitrary time slot with probability of $p$. Hence,

$$
\begin{aligned}
p_{s}(N) & =\mathrm{P}\{\text { transmission is succesful }\} \\
& =\mathrm{P}\{N-1 \text { other stations do not transmit }\}=(1-p)^{N-1} .
\end{aligned}
$$

When $\rho=N p$ is kept constant while $N \rightarrow \infty$ and $p \rightarrow 0$, we get,

$$
p_{s}(\infty)=\lim _{N \rightarrow \infty}(1-\rho / N)^{N-1}=\lim _{N \rightarrow \infty} \frac{(1-\rho / N)^{N}}{1-\rho / N}
$$

and using (3) gives,

$$
p_{s}(\infty)=e^{-\rho} .
$$

Hence, the throughput is,

$$
\theta(\rho)=\rho e^{-\rho} .
$$

Taking the first derivate in respect to $\rho$ gives,

$$
\theta^{\prime}(\rho)=e^{-\rho}-\rho e^{-\rho}=(1-\rho) e^{-\rho},
$$

which is positive when $\rho \in[0,1)$ and zero at $\rho=1$. Thus, the maximum throughput (per wavelength) is,

$$
\theta(1)=1 / e \approx 0.3679 .
$$

b) Tell-and-Go:

Briefly, $N-1$ stations can try to transmit to an arbitrary station, say station 1 . So each station $j, j \neq 1$, transmits with probability of $p \cdot \frac{1}{N-1}$ to station 1 at each time slot. The probability of the complement case, i.e. station does not transmit, is

$$
1-\frac{p}{N-1} .
$$

The probability that at least one station transmits to station 1 is clearly,

$$
\begin{aligned}
p_{t}(N) & =1-\mathrm{P}\{\text { no one transmits to station } 1\} \\
& =1-\left(1-\frac{p}{N-1}\right)^{N-1}
\end{aligned}
$$

As $N \rightarrow \infty$, i.e. $N-1 \rightarrow \infty$ we get using (3),

$$
p_{t}(\infty)=1-e^{-p}
$$

Hence, the throughput per receiver is

$$
\theta(p)=1-e^{-p}
$$

The first derivate is,

$$
\theta^{\prime}(p)=e^{-p}
$$

which is clearly always positive when $p \in[0,1]$, and thus the maximum is reached when $p=1$, and is,

$$
\theta(1)=1-e^{-1} \approx 0.6321
$$

The constant $e$ is defined as the limit,

$$
\begin{equation*}
e=\lim _{n \rightarrow \infty}\left(1+\frac{1}{n}\right)^{n} \tag{1}
\end{equation*}
$$

i)

$$
\begin{align*}
\lim _{n \rightarrow \infty}\left(1-\frac{1}{n}\right)^{n} & =\lim _{n \rightarrow \infty}\left(\frac{n-1}{n}\right)^{n}=\lim _{n \rightarrow \infty} \frac{1}{\left(\frac{n}{n-1}\right)^{n}} \\
& =\lim _{k \rightarrow \infty} \frac{1}{\left(\frac{k+1}{k}\right)^{k+1}}=\lim _{k \rightarrow \infty} \frac{1}{\left(1+\frac{1}{k}\right)^{k}\left(1+\frac{1}{k}\right)} \quad=1 / e \tag{2}
\end{align*}
$$

ii)

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left(1-\frac{a}{n}\right)^{n}=\lim _{n \rightarrow \infty}\left[\left(1-\frac{1}{n / a}\right)^{n / a}\right]^{a}=[1 / e]^{a}=e^{-a} . \tag{3}
\end{equation*}
$$

2. Wavelength Routing Networks.
a) Consider the bidirectional ring WRN consisting of 7 WSXC's and 7 non-blocking NAS's depicted in slide 39 of lecture 9 . Solve the routing and channel assignment problem that realizes the full logical/optical connectivity between the 7 stations using 6 wavelengths.
b) Consider the multistar WRN consisting of 7 WSXC's and 7 non-blocking NAS's depicted in slide 39 of lecture 9 . Solve the routing and channel assignment problem that realizes the full logical/optical connectivity between the 7 stations using only 2 wavelengths.

## Solution:

## a) Bidirectional ring WRN:



Figure 15: One feasible solution for full connectivity.
The feasible routing is straightforward. One can, for example, first assign the longest connections ( 3 hops), which are drawn in the outer rings in the figure. That requires at minimum 3 wavelengths already. Then configuring the rest of the connections requires additional 3 wavelengths leading to total 6 wavelengths. This is clearly an optimal configuration as the shortest paths are used and there is no unused capacity.
b) Multistar:

This problem can be divided to 7 independent problems. Namely, we can first consider only connections routed via WSXC $A$, as none of them uses same link with any connection routed via some other WSXC. The same holds for any WSXC. The wavelength assignment problem is depicted in Fig. (17) for WSXC $A$ and $B$. The resulting ring can be clearly colored with two colours, and hence wavelength assignment with 2 wavelengths is possible. The same holds for any WSXC.


Figure 16: A multistar topology.


Through Node B


Figure 17: Wavelength assignment problem illustrated for routes routed via WSXC $A$ and $B$.

## 3. Linear Lightwave Networks.

a) Consider the mesh LLN consisting of 5 waveband selective LDC's and 7 nonblocking NAS's depicted in slide 39 of lecture 9 . Solve the routing and channel assignment problem that realizes the logical connection hypergraph depicted in slide 42 of lecture 9 using 2 wavebands, 3 wavelengths per waveband, and TDM/TWDMA in FT-TR mode.
b) Consider the multistar LLN consisting of 7 waveband selective LDC's and 7 nonblocking NAS's depicted in slide 39 of lecture 9 . Solve the routing and channel assignment problem that realizes the logical connection hypergraph depicted in slide 42 of lecture 9 using only 1 waveband, 3 wavelengths per waveband, and TDM/TWDMA in FT-TR mode.

## Solution:



Figure 18: The physical mesh topology and LCH.
a) Mesh physical topology:

The stations in LCH we divided in the following way:

| $E 1:$ | $2,5,7$ | $E 5: 1,3,7$ |
| :--- | :--- | :--- |
| $E 2:$ | $1,2,6$ | $E 6: 4,6,7$ |
| $E 3:$ | $2,3,4$ | $E 7: 3,5,6$ |
| $E 4:$ | $1,4,5$ |  |

The used waveband routing is illustrated in the Fig. 19. Wavebands $E 4$ and $E 6$, as well as $E 5$ and $E 7$, share the same links (in fact same route!) and must be assigned thus different wavebands. Hence, a feasible waveband assignment could be,

$$
\mathrm{WB}_{1}=\{E 1, E 2, E 3, E 4, E 5\}
$$

and,

$$
\mathrm{WB}_{2}=\{E 6, E 7\} .
$$



Figure 19: The waveband routing.

In each waveband there are 3 stations, so each station can have unique wavelength $\lambda_{i}$ among the 3 wavelengths available per waveband. As there are two receivers, the time division multiplexing halves the capacity between the stations within same waveband.
Note that each station $j$ must have 3 transceivers, one for each subnet $E_{i}$. Each transceiver then operates in FT-TR mode.
b) Multistar physical topology:

As can be seen from the figure, the mapping between physical topology and LCH is obvious: each ONN (LDC in this case) forms own subnet and thus one waveband is sufficient.
The problem is essentially the same case as in problem $7 / 2$, but here each transmitter is using fixed wavelength. Thus, for example station 1 operating at subnetwork $E_{2}$ must use the same wavelength to transmit to stations 2 and 6 . Hence, 3 wavelengths are needed.


Figure 20: Multistar.


[^0]:    ${ }^{1}$ Four color theorem states that any planar graph (or map) can be colored with 4 colors so that no neighbouring nodes have the same color. The problem was originally proposed in 1852 and only 1976 Haken and Appel finally managed to prove it correct with computer assistance.

[^1]:    ${ }^{2}$ clique is a fully connected sub graph, and hence each node needs unique color

