Pricing Napster and Other Internet Peer-to-Peer Applications

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Introduction

Of the many services provided on the Internet, a newly emerging service seems to be peer-to-peer file sharing. Peer-to-peer technology enables users to connect to the personal computer of others. One particular application is the providing of an index of MP3 files, making searching for files and connecting to other people's PCs easier.

Some issues being raised about such services are security threats, privacy concerns, and copyright issues. The threats to security and privacy come from the potential to access restricted parts of the user's hard drive, and gain access to private and confidential data. Also, there is much uncertainty about how pricing and cost recovery can be structured for such systems.

In this paper, we suggest the use of incentives using a principal agent framework to create a payment scheme for Internet-related peer-to-peer services. We suggest that the use of incentives in the payment scheme may provide users with better service and reduce some of the concerns. Finally, we raise the point that users themselves also provide a service for Napster as well as other users by providing access to certain parts of their hard drives. Therefore, any pricing model should take into consideration the users as a service providers as well.

Principal Agent Model

Principal agent theory helps to identify the conditions under which the interests of a party, the principal, will be realized through establishment of a relationship with another party, the agent (Ross 1973, Bowie and Freeman 1992). Principal agent relationships exist when the principal wants a task done, service provided, or output produced and for a variety of reasons, relies on someone else (the agent) to complete the task, provide the service or produce the output (Holmstrom 1979). A principal agent problem arises when agents have hidden information about the outcome, are risk averse, or it is not easy for the principal to monitor the agent. Principal agent theory seeks to understand real life problems of loss of control, information asymmetry, monitoring costs, and conflicting goals in organizations. Essentially, principal agent theory suggests that both the principal and the agent exhibit self-interest behaviors.

To enhance sharing of goals with the agent, the principal can enforce certain conditions through a contract, or incentive scheme. The principal’s goal is to devise an incentive scheme under which the agent will not shirk. At a minimum, the incentive scheme should satisfy two conditions. First, the agent must be willing to accept the incentive scheme. In other words, the incentive scheme must offer the agent at least as much utility (satisfaction) as the agent’s next best alternative, or threshold wage, also known as the agent's reservation utility. This reservation utility is the compensation that the agent could receive by performing some other task rather than work for the principal. If the agent is not offered at least this amount, the agent will prefer to work for someone else. Second, the scheme should induce the agent to provide the level of effort that the principal desires.

The principal (P) wants to hire an agent (A) to perform a service. The output (x) depends on the agent's level of effort (a), and a random factor that the agent does not control. In the context of Internet peer-to-peer technology the random factor may cover a number of things such as a temporary shutdown of some Internet servers, connection problems due to heavy bandwidth, peers intentionally logging off and thus severing the connection, etc. The principal's goal is to devise an incentive scheme (I) that maximizes his utility (U_P) while ensuring that the agent receives a utility (U_A) that is at least as much as the agent's reservation utility (threshold wage) K. Thus, following Holmstrom (1979) the principal's problem becomes:

$$\max E\{U_P(x-I(x))\} \tag{1}$$

$$\text{subject to } E\{U_A(I(x)\} \geq K \tag{2}$$

$$\text{subject to } E\{U_A(I(x), a\} \geq E\{U_A(I(x), a'\} \text{ for all } a' \neq a \tag{3}$$

This says that (1) the principal wants to choose an incentive scheme I(x) to maximize her expected utility, subject to (2) the agent's utility must be at least as much as his reservation utility (or threshold wage), and to (3) effort level a is more desirable (results in at least as much payment) to the agent than any other effort level.

Application to Napster

One of the most popular peer-to-peer services now available is Napster. Napster (http://www.napster.com) acts like a music search engine, maintaining servers that contain directories of music files kept by registered users (Kover 2000). The music files themselves remain on the user's hard drive. Napster's software provides users with a list of active users that possess the files they are looking for, provides a connection between the cooperating users' computers, and transfers the desired file. This service is currently free and has not paid copyrighted-related royalties. The popularity of this and similar services has alarmed many elements of the music industry and has led to them suing the company for violating copyright laws (http://www.riaa.com/Napster_legal.cfm). In addition to these intellectual property issues, the security of computers registered with Napster and the privacy of information in them is also of concern.

Some possible principal agent relationships in this area are between the Internet user and Napster, recording artists and Napster, and the music industry and Napster. These are multiple principal single agent relationships. In the first relationship, users (acting as principals) desire to find and download MP3 music files. Napster (the agent) provides a service by maintaining an index of MP3 databases. On the other hand, we can also view the relationship as users (acting as principals) providing hard-drive access to Napster (the principal) or any other user via Napster. Additionally, if recording artists or others in the music industry (principals) decided to use Napster-like services as a medium to sell their music or other goods or services, Napster could act as an agent providing marketing services as well.

Napster currently receives no fee for its services. Potential investors have suggested that it collect subscription fees from members, persuade record labels to use the service as a marketing tool, or act as an e-commerce outlet for CDs (Harmon 2000, Kover 2000). In October of 2000, Bertelsmann, the German media and publishing firm formed a strategic alliance with Napster to commercialize Napster's file swapping services (King 2000). Edel Music, the European independent music label has also agreed to distribute music using Napster's service (Boston 2001). The service is expected to be in the form of a monthly subscription. Although there is no indication of what the fee will be, they say that according to a survey of 20,000 Napster users by Webnoize (http://www.webnoize.com), a large majority of users are willing to pay up to $15.00 monthly for the service (Evers 2001). Subscription-based fees are popular in many areas. Emusic.com, cable TV, and ISP membership fee services are just a few examples. But incentive-based pricing schemes tied closely to the agent's effort (where the effort includes that required to maintain security and privacy, and protect copyrights) may provide an alternative to subscription-based plans, and are more likely to ensure that the agent will provide an effort level required to maintain the principals' desired level of service.

In the principal agent paradigm, the principal's goal is to provide an incentive scheme under which the agent will not shirk. In other words, the users want Napster to maintain a properly updated index (without broken links, flaky or malicious computers, etc). Napster or another service could maintain a list of only those files that have a copyright protection mechanism (which have proven in the past to be wildly unpopular) or could exert more efforts to preserving the security and privacy of the principals' computer systems (for example, ensuring that the files are virus free).

If we assume that Napster, the agent, is (somewhat) risk-averse (i.e. prefers a "sure thing" to a bet of equal value), the principals can offer an incentive scheme with a risk premium that provides enough incentives so that the agent does not shirk and accepts a certain degree of risk.

For the various relationships, the output and effort level can be defined as follows:

When the principal is the Internet user, the agent's output x can be the money that the user saves by downloading music files from Napster rather than buying CDs or tapes or other traditional physical embodiments of the music. The agent's level of effort a can be defined as the effort required for maintaining the index of files (in terms of manpower, equipment, etc) in monetary values.

In other relationships, the recording artist (or record companies in their role as copyright holders) can offer an incentive scheme that pays according to the number of times their music is searched for and downloaded.

As an example, consider the problem facing a risk-neutral Internet user of employing the risk-averse Napster to provide an indexing service. The decision variable is the incentive scheme that will be offered to Napster. We will make some assumptions about the exogenous variables – those that are not determined by the actions of the principal. For simplicity, assume that Napster can either exert a high level of effort (a = 1) or a low level of effort (a = 0). We can define a high effort level (a = 1) as effort required to maintain the index properly in addition to providing security, privacy and copyright protection. Low effort level (a = 0) can be defined as the minimum effort required to maintain the index. In this example, let the outcome be x = 1 if the desired file is found and downloaded with traditional security, privacy, and copyright protection, and x = 0 if it is not. In this example, effort level and outcome are not dollar values, but binary values. Of course, this example can be expanded to include more than two levels of effort and outcome, making the model more realistic. For example, outcome can be high, medium or low. High outcome can be when files are downloaded with traditional copyright, security and privacy protection, medium outcome can describe situations where files are downloaded without these protections, and low outcome can be when files cannot be downloaded. To make the model even more realistic, outcome x can be a continuous random variable that represents user profit (or savings) in dollar values. These other models may be developed later, but aren't necessary to make the point of this paper, about the utility of principal agent theory for describing certain Internet peer-to-peer transactions. For the scope of this paper, we will limit our effort and outcome to binary values.

Even with the agent exerting maximum effort, random factors outside his control (as described above) affect the user's ability to download successfully. Thus we assume that the probability of successfully downloading the file, given the agent's effort level, is as shown in Table 1.

Table 1. Outcome Probability

		Outcome (x)	
		0	1
Effort (a)	0	**0.5**	**0.5**
	1	**0.25**	**0.75**

Since the outcome does depend on the effort of Napster, but this effort is not directly observable by the user, the user will offer an incentive scheme based on the outcome. The incentive scheme is then depicted as I(x).

What is the output of the agent, given the above outcomes? The output f(x) is the monetary savings per file, when downloading via Napster, which can be seen as the output created by Napster. Limiting our discussion to compact discs (CDs) for simplicity, if we assume that one CD costs $15, and there are approximately 15 songs in one CD, the Internet user stands to save the expense of 1 (dollar) for each file they download rather than buying in the traditional manner a CD containing the desired file. Bestbuy and Emusic.com also charge approximately $0.99 (≈ $1.00) per MP3 file, so the assumption that the user stands to make a profit of $1.00 per file is not unreasonable. Additionally, the user will stand to make a savings of $0 if the file is not downloaded properly (perhaps even less if the file contains a virus and ends up infecting the user's computer). Thus, when outcome x = 1 (file is downloaded successfully), the agent's output f(x=1) can be seen as $1.00. On the other hand, when outcome x = 0, the agent's output f(x=0) is $0.00. The Internet user's net profit is then this profit minus what Napster charges him: f(x) – I(x).

Since we assume that the user is risk-neutral (i.e. the user has a linear utility function), the user's utility equals the output minus what Napster charges him: $U_P(x) = x - I(x)$. Since Napster is risk-averse, it has a concave utility function. Also, Napster has a disutility for effort. Thus Napster's utility equals the utility it gets from the incentive scheme minus the disutility it gets from providing effort e: $U_A(x) – V(a)$. Here, we are using a special case of a class of functions known as power functions to describe

Napster's risk-averse utility. Power functions take the form $U = BX^{\alpha}/\alpha$. In this example, we use $B = 0.5$, $X = I(x)$, $\alpha = 0.5$[1]. Thus $U = 0.5I(x)^{0.5}/0.5 = \sqrt{(I(x))}$. If we assume that it costs Napster 20 cents to provide a unit of effort per file (0.20a), then Napster's total utility (the utility from its net profit, which would be the incentive scheme minus the cost of effort for that one file), $U_A(x) – V(a)$ can be represented as $\sqrt{(I(x))} – 0.20a$.

Our last assumption is about Napster's reservation utility K. Although there have been suggestions that Napster display banner ads or accept subscription fees to make money, these schemes are not useful in this model. For instance, a subscription fee is a fixed fee, and depending on the user, for the same fixed fee, one user may be able to download thousands of files, whereas another user may only use Napster a handful of times. Then, comparing our per-file incentive scheme to a subscription fee is not suitable. For similar reasons, using banner fees as the reservation utility is also not acceptable. For this reason, let us model the current situation of Napster, where it receives no fees for its service. In this case, the reservation utility is zero (K = 0). At first glance, it may seem that using a zero value for the reservation utility will not provide Napster with an alternative incentive scheme. But the constraints of the model (such as ensuring that the agent's utility for the desired effort level be at least as much as any other effort level) allow us to obtain a positive solution for the incentive scheme.

Table 2 summarizes our assumptions up to this point.

[1] As α decreases, risk aversion increases (Besanko et al. 1996).

Table 2. Summary of Variables

Variables	Values	Description
Effort Level a	High (a = 1)	Effort to maintain index and provide security, privacy and copyright protection
	Low (a = 0)	Minimum effort required to maintain index
Output(x)	Successful (x = 1)	File is downloaded with security, privacy and copyright protection
	Unsuccessful (x = 0)	File is not downloaded properly
Outcome f(x)	f(x=1) = 1 (dollar)	When file is downloaded successfully, the user stands to make a dollar profit
	f(x=0) = 0 (dollar)	When file is not downloaded successfully, user makes no profit.
Cost of Effort	0.20a	Each unit of effort costs 20 cents/file
Agent's Utility	$\sqrt{(I(x))} - 0.2a$	The agent's utility is the utility received from his proceeds, minus the cost of effort
Reservation Utility K	0	Napster's current reservation utility is zero, since it does not receive fees from its users.

Given all this, we can now formulate the Internet user – Napster relationship as a specific instance of principal agent theory, where the principal's problem becomes:

$$\text{maximize } E\{f(x) - I(x)\} \qquad (4)$$

$$\text{subject to } E\{\sqrt{(I(x)\,)} - 0.2a\} \geq 0 \qquad (5)$$

$$\text{subject to } E\{\sqrt{(I(x)\,)} - 0.2a\} \geq E\{\sqrt{(I(x)\,)} - 0.2a'\} \text{ for all } a' \neq a \qquad (6)$$

To this, we also add the conditions that Napster's utility is not negative:

$$\sqrt{(I(x)} \geq 0 \text{ for all } x \qquad (7)$$

This says that the Internet user wants to (4) maximize his utility which is the money he saves from downloading a file (as opposed to buying) minus the payment to Napster, subject to the constraints that (5) Napster's utility should be at least as much as the utility it currently receives (nothing), and (6) exerting effort level a will yield at least as much utility to Napster as any other level. Lastly, (7) ensures that the agent's utility for profits cannot be negative.

If a high level of effort (a = 1) is desired, we can use the probabilities from Table 1 to describe the specific problem:

$$\max\{(1 - I(1))* 0.75 + (0-I(0))*0.25\} \quad (8)$$

$$\text{subject to } (\sqrt{(I(1))} - 0.2)*0.75 + (\sqrt{(I(0))} - 0.2)*0.25 \geq 0 \quad (9)$$

$$\text{subject to } (\sqrt{(I(1))} - 0.2)*0.75 + (\sqrt{(I(0))} - 0.2)*0.25$$

$$\geq (\sqrt{(I(1))})*0.5 + (\sqrt{(I(0))})*0.5 \quad (10)$$

$$\text{subject to } \sqrt{(I(1)} \geq 0, \text{ and } \sqrt{(I(0)} \geq 0 \quad (11)$$

In particular, (8) says that the principal's expected utility for a high effort level (a = 1) should be maximized, subject to the constraints that (9) Napster's expected utility is at least as much as that of its current fee, (10) exerting a high level of effort (a = 1) should provide Napster with at least as much utility as exerting a low level of effort (a = 0), and (11) Napster's utility is nonnegative.

This maximum is reached when I(1) = 0.64 and I(0) = 0. Furthermore, the principal's expected utility, Napster's expected utility when providing a high effort level (a = 1), and Napster's expected utility when providing a low effort level (a = 0) are respectively $E(U_P) = 0.27$, $E(U_A(a=1)) = 0.4$, and $E(U_A(a=0)) = 0.4$. For a detailed description of how these values were obtained, please refer to Appendix A.

Thus, for every file downloaded, the Internet user pays 64 cents to Napster if it is successful and 0 cents if it is not. This gives the user an expected utility (or expected net savings) of 27 cents. Napster receives an expected utility of 4 cents which is an improvement over the zero utility it presently receives. Also, the agent's utility for the two effort levels is equal. In other words, the agent is indifferent between the two actions and their associated rewards. In such cases, we can assume, as do principal agent theorists, that factors outside of the principal agent paradigm (such as reputation and ethical values) will induce the agent to select the action more favored by the principal (Bowie and Freeman 1992).

Discussion

When Napster and Bertelsmann begin to offer a subscription-based service, it should be recognized that users are now paying for a service which they in part provide. After all, Napster only acts as a huge music index, and it is the users that are, in a sense, opening up their directories and allowing Napster (and other users via Napster) to download their files. In this sense, users are providing a service – not the offering of copyrighted materials, but the use of their part of the huge "database in cyberspace". In this sense it is the user that is acting as an agent for Napster, by providing them access to parts of their computers. Then, users should realize that they could also charge Napster a fee for this service. After all, without access to users' files and directories Napster-like services could not exist. In this case, the user as the agent provides effort levels that could include keeping the files virus-free, providing appropriate security and privacy, and providing a constant connection. In fact, we could use the same model we used to obtain Napster's incentive scheme to model the incentive scheme for a user. Expanded further, this could be viewed as a problem of one principal (Napster), choosing from many agents (users). Napster (or another user via Napster) would choose from the available pool of users (and their files) depending on things such as user reputation and ability (e.g. how reputable is the user, is he known to have virus-free files, is he knowledgeable in security-related matters), making the model even more interesting.

If we go one step further, we can model the relationship between Napster the agent, and the user, the principal, when Napster receives a subscription fee, and the principal receives a small fee whenever another user downloads one of his files via Napster.

The incentive scheme per file suggested in this research, while providing better motive for agents to work on behalf of the principal's interest may not be a suitable solution for current Internet-based peer-to-peer schemes with the current payment schemes available. For example, the user would have to provide their credit card information every time they downloaded a file. However, variations of the method could work with current schemes. For example, a user could pay a monthly subscription fee, use Napster to download music files, then at the end of the month, compare the subscription fee to the per file incentive scheme, and when renewing the monthly subscription, only pay the difference of the two. In the future, when widespread use of electronic payment schemes are available, the research suggests that incentive schemes will be easier to use and will therefore be suggested more frequently.

Conclusion

Internet-related peer-to-peer services may become very widespread, but there are concerns regarding security and copyright issues. Price models for these services have not yet been thoroughly explored though several have been suggested. This paper introduced the use of incentive schemes as a means for these service providers to receive payment for their services while simultaneously reducing the risks related to security, privacy, and copyright issues.

The results of the paper shows that Napster can receive a higher utility with an incentive-based service than that which it currently receives, while being more motivated to provide the users with a higher level of security and privacy.

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Appendix A. Derivation of the Model

Optimization problems with constraints can be solved using the Lagrange Multiplier method. The Lagrange Multiplier method is used to find the extremum of $f(\chi_1, \chi_2, \ldots, \chi_n)$ subject to the constraints $g_j(\chi_1, \chi_2, \ldots, \chi_n) \leq C$ where j = 1..m (inequality constraints), and $\chi_i \geq 0$ for all i = 1..n (non-negativity constraints), where f and g are functions with continuous partial derivatives. For a more detailed discussion of the Lagrangian Multiplier method, see Huang and Crooke (1997).

For an extremum for f to exist with the given inequality and non-negativity constraints where the Lagrangian function L is defined as

$$L = f(\chi_1, \chi_2, \ldots, \chi_n) - \Sigma\mu_j(g_j(\chi_1, \chi_2, \ldots, \chi_n) - c_j))$$

and μ_j are the Lagrangian multipliers, the following Kuhn-Tucker conditions should be satisfied:

$$\partial L/\partial\chi_i \leq 0,\ \chi_i \geq 0, \text{ and } \chi_i\, \partial L/\partial\chi_i = 0 \text{ (for all i = 1…n)}$$

$$\partial L/\partial\mu_j \leq 0,\ \mu_j \geq 0, \text{ and } \mu_j\, \partial L/\partial\mu_j = 0 \text{ (for all j = 1…m)}$$

The stationary point ($\chi_1 = x_1, \chi_2 = x_2, \ldots, \chi_n = x_n$) that satisfies the above conditions is the point where $f(\chi_1, \chi_2, \ldots, \chi_n)$ is an extremum under the given constraint.

This method can be extended to more than one constraint by adding additional Lagrangian multipliers as needed.

In our principal agent problem involving Napster, the principal's problem was generally stated as:

$$\max E\{f(x) - I(x)\} \qquad (4)$$

subject to $E\{\sqrt{}(I(x)) - 0.2a\} \geq 0$ (5)

subject to $E\{\sqrt{}(I(x)) - 0.2a\} \geq E\{\sqrt{}(I(x)) - 0.2a'\}$ for all $a' \neq a$ (6)

subject to $\sqrt{}(I(x) \geq 0$ (7)

To induce a high level (a=1) of effort, the problem is:

$\max\{(1 - I(1))* 0.75 + (0-I(0))*0.25\}$ (8)

subject to $(\sqrt{}(I(1)) - 0.2)*0.75 + (\sqrt{}(I(0)) - 0.2)*0.25 \geq 0$ (9)

subject to $(\sqrt{}(I(1)) - 0.2)*0.75 + (\sqrt{}(I(0)) - 0.2)*0.25$

$\geq (\sqrt{}(I(1))*0.5 + (\sqrt{}(I(0)))*0.5$ (10)

subject to $\sqrt{}(I(1) \geq 0, \sqrt{}(I(0) \geq 0$ (11)

These can be further simplified as follows:

$\max (0.75(1 - I(1)) - 0.25I(0))$ (8a)

subject to $0.75\sqrt{I(1)} + 0.25\sqrt{I(0)} - 0.2 \geq 0$ (9a)

subject to $0.25\sqrt{I(1)} - 0.25\sqrt{I(0)} - 0.2 \geq 0$ (10a)

subject to $\sqrt{}(I(1) \geq 0, \sqrt{}(I(0) \geq 0$ (11a)

Formulas (8a) through (11a) can be seen as an optimization problem with two variables I(1) and I(0). For readability, let us substitute χ_1 for $\sqrt{I(1)}$ and χ_0 for $\sqrt{I(0)}$. Using the Lagrangian Multiplier method, (8a) through (11a) becomes:

$\max f(\chi_1, \chi_0) = 0.75(1 - \chi_1^2) - 0.25\chi_0^2$ (8b)

subject to $g_1(\chi_1, \chi_0) = -0.75\chi_1 - 0.25\chi_0 + 0.2 \leq 0$ (9b)

subject to $g_2(\chi_1, \chi_0) = -0.25\chi_1 + 0.25\chi_0 + 0.2 \leq 0$ (10b)

subject to $\chi_1 \geq 0, \chi_0 \geq 0$ (11b)

Letting μ_1 and μ_2 denote the Lagrangian multipliers, the Lagrangian function becomes:

$$L(\chi_1, \chi_0, \mu_1, \mu_2) = f(\chi_1, \chi_0) - \mu_1 g_1(\chi_1, \chi_0) - \mu_2 g_2(\chi_1, \chi_0)$$

$$= 0.75(1 - \chi_1^2) - 0.25\chi_0^2 + \mu_1(0.75\chi_1 + 0.25\chi_0 - 0.2) + \mu_2(0.25\chi_1 - 0.25\chi_0 - 0.2)$$

$$= -0.75\chi_1^2 - 0.25\chi_0^2 + 0.75 + \mu_1(0.75\chi_1 + 0.25\chi_0 - 0.2) + \mu_2(0.25\chi_1 - 0.25\chi_0 - 0.2)$$

The first partial derivatives of the Lagrangian are:

$$\partial L/\partial\chi_1 = -1.5\chi_1 + 0.75\mu_1 + 0.25\mu_2 = 0$$

$$\partial L/\partial\chi_0 = -0.5\chi_0 + 0.25\mu_1 - 0.25\mu_2 = 0$$

$$\partial L/\partial\mu_1 = 0.75\chi_1 + 0.25\chi_0 - 0.2 = 0$$

$$\partial L/\partial\mu_2 = 0.25\chi_1 - 0.25\chi_0 - 0.2 = 0$$

To find the point where $f(\chi_1, \chi_0)$ is maximum, we need to find a point where the following Kuhn-Tucker conditions are satisfied:

$$\partial L/\partial\chi_1 \leq 0,\ \chi_1 \geq 0, \text{ and } \chi_1\, \partial L/\partial\chi_1 = \chi_1(-1.5\chi_1 + 0.75\mu_1 + 0.25\mu_2) = 0 \quad (12)$$

$$\partial L/\partial\chi_0 \leq 0,\ \chi_0 \geq 0, \text{ and } \chi_0\, \partial L/\partial\chi_0 = \chi_0(-0.5\chi_0 + 0.25\mu_1 - 0.25\mu_2) = 0 \quad (13)$$

$$\partial L/\partial\mu_1 \geq 0,\ \mu_1 \geq 0, \text{ and } \mu_1\, \partial L/\partial\mu_1 = \mu_1(0.75\chi_1 + 0.25\chi_0 - 0.2) = 0 \quad (14)$$

$$\partial L/\partial\mu_2 \geq 0,\ \mu_2 \geq 0, \text{ and } \mu_2\, \partial L/\partial\mu_2 = \mu_2(0.25\chi_1 - 0.25\chi_0 - 0.2) = 0 \quad (15)$$

Since there are four variables ($\chi_1, \chi_0, \mu_1, \mu_2$) of which either that variable or its derivative can equal zero, there are 2^4 cases to consider. The cases to consider are shown in Table 3.

Table A1. Possible Cases for the Kuhn-Tucker Conditions

Case	μ_1, μ_2	χ_1, χ_0
1	$\mu_1 = 0$, $\mu_2 = 0$	$\chi_1 = 0$, $\chi_0 = 0$
2	$\mu_1 = 0$, $\mu_2 = 0$	$\chi_1 = 0$, $\partial L/\partial\chi_0 = 0$
3	$\mu_1 = 0$, $\mu_2 = 0$	$\partial L/\partial\chi_1 = 0$, $\chi_0 = 0$
4	$\mu_1 = 0$, $\mu_2 = 0$	$\partial L/\partial\chi_1 = 0$, $\partial L/\partial\chi_0 = 0$
5	$\mu_1 = 0$, $\partial L/\partial\mu_2 = 0$	$\chi_1 = 0$, $\chi_0 = 0$
6	$\mu_1 = 0$, $\partial L/\partial\mu_2 = 0$	$\chi_1 = 0$, $\partial L/\partial\chi_0 = 0$
7	$\mu_1 = 0$, $\partial L/\partial\mu_2 = 0$	$\partial L/\partial\chi_1 = 0$, $\chi_0 = 0$
8	$\mu_1 = 0$, $\partial L/\partial\mu_2 = 0$	$\partial L/\partial\chi_1 = 0$, $\partial L/\partial\chi_0 = 0$
9	$\partial L/\partial\mu_1 = 0$, $\mu_2 = 0$	$\chi_1 = 0$, $\chi_0 = 0$
10	$\partial L/\partial\mu_1 = 0$, $\mu_2 = 0$	$\chi_1 = 0$, $\partial L/\partial\chi_0 = 0$
11	$\partial L/\partial\mu_1 = 0$, $\mu_2 = 0$	$\partial L/\partial\chi_1 = 0$, $\chi_0 = 0$
12	$\partial L/\partial\mu_1 = 0$, $\mu_2 = 0$	$\partial L/\partial\chi_1 = 0$, $\partial L/\partial\chi_0 = 0$
13	$\partial L/\partial\mu_1 = 0$, $\partial L/\partial\mu_2 = 0$	$\chi_1 = 0$, $\chi_0 = 0$
14	$\partial L/\partial\mu_1 = 0$, $\partial L/\partial\mu_2 = 0$	$\chi_1 = 0$, $\partial L/\partial\chi_0 = 0$
15	$\partial L/\partial\mu_1 = 0$, $\partial L/\partial\mu_2 = 0$	$\partial L/\partial\chi_1 = 0$, $\chi_0 = 0$
16	$\partial L/\partial\mu_1 = 0$, $\partial L/\partial\mu_2 = 0$	$\partial L/\partial\chi_1 = 0$, $\partial L/\partial\chi_0 = 0$

We can reduce some of the cases we need to examine by noting that the particular Kuhn-Tucker condition from above:

$$\partial L/\partial\mu_2 \geq 0,\ \mu_2 \geq 0, \text{ and } \mu_2\, \partial L/\partial\mu_2 = \mu_2 (0.25\chi_1 - 0.25\chi_0 - 0.2) = 0 \qquad (15)$$

cannot be satisfied when $\chi_1 = 0$. When $\chi_1 = 0$,

$$\partial L/\partial\mu_2 = 0.25\chi_1 - 0.25\chi_0 - 0.2 = -0.25\chi_0 - 0.2 < 0$$

and this contradicts the condition that $\partial L/\partial\mu_2 \geq 0$. Therefore we only need consider the cases in which $\partial L/\partial\chi_1 = 0$ (specifically, cases 3, 4, 7, 8, 11, 12, 15, and 16). Of these cases, the only one that provides a solution while satisfying the Kuhn-Tucker conditions is case 7. We show case 7 as follows:

Case 7 $\mu_1 = 0, \partial L/\partial\mu_2 = 0, \partial L/\partial\chi_1 = 0, \chi_0 = 0$

Since $\mu_1 = 0$ and $\chi_0 = 0$:

$$\partial L/\partial\chi_1 = -1.5\chi_1 + 0.75\mu_1 + 0.25\mu_2 = -1.5\chi_1 + 0.25\mu_2 = 0$$

$$\Leftrightarrow \mu_2 = 6\chi_1 \qquad (16)$$

$$\partial L/\partial\mu_2 = 0.25\chi_1 - 0.25\chi_0 - 0.2 = 0.25\chi_1 - 0.2 = 0$$

$$\Leftrightarrow \chi_1 = 0.8 \qquad (17)$$

Substituting this value into (16) gives us $\mu_2 = 4.8$.

To ensure that these values satisfy the Kuhn-Tucker conditions, we must make sure that $\partial L/\partial\chi_0 \leq 0$ and $\partial L/\partial\mu_1 \geq 0$:

$$\partial L/\partial\chi_0 = -0.5\chi_0 + 0.25\mu_1 - 0.25\mu_2 = -0.25\mu_2 < 0$$

$$\partial L/\partial\mu_1 = 0.75\chi_1 + 0.25\chi_0 - 0.2 = 0.75\chi_1 - 0.2 = 0.75(0.8) - 0.2 = 0.4 > 0$$

Therefore the quadruple set $(\mu_1, \mu_2, \chi_1, \chi_0) = (0, 4.8, 0.8, 0)$ satisfy the Kuhn-Tucker conditions and provides the maximum for this model. Remembering that we substituted χ_1, and χ_0 for $\sqrt{I(1)}$ and $\sqrt{I(0)}$, respectively, the incentive scheme (I(1), I(0)) is $(0.8^2, 0) = (6.40, 0.00)$.

The expected utility of the principal when Napster provides a high level of effort is now:

$$E(U_P(e=1)) = E\{x - I(x)\} = (1 - I(1))*0.75 + (0 - I(0))*0.25$$
$$= (1 - 0.64)*0.75 + 0*0.25 = 0.27$$

The expected utility of the agent when he provides a high level of effort is:

$$E(U_A(e=1)) = E\{\sqrt{(I(x))} - 0.2e\} = (\sqrt{(I(1)} - 0.2)*0.75 + (\sqrt{(I(0)} - 0.2)*0.25$$
$$= (0.8 - 0.2)*0.75 + (0 - 0.2)*0.25 = 0.4$$

When Napster provides a low level of effort with the given incentive scheme, its expected utility is:

$$E(U_A(e=0)) = E\{\sqrt{}(I(x)) - 0.2e\} = \sqrt{}(I(1)*0.5 + \sqrt{}(I(0)*0.5$$

$$= (0.8)*0.5 + 0*0.5 = 0.4$$

In this model, with the given assumptions, the agent's expected utility for a high level of effort and a low level of effort are identical.

Let us briefly take a look at why the other cases do not give an optimum value set for the model. For example, take case 12:

Case 12 $\partial L/\partial\mu_1 = 0, \mu_2 = 0, \partial L/\partial\chi_1 = 0, \partial L/\partial\chi_0 = 0$

Since $\mu_2 = 0$,

$$\partial L/\partial\chi_1 = -1.5\chi_1 + 0.75\mu_1 + 0.25\mu_2 = -1.5\chi_1 + 0.75\mu_1 = 0$$

$$\Leftrightarrow \chi_1 = 0.5\mu_1 \quad (18)$$

$$\partial L/\partial\chi_0 = -0.5\chi_0 + 0.25\mu_1 - 0.25\mu_2 = -0.5\chi_0 + 0.25\mu_1$$

$$\Leftrightarrow \chi_0 = 0.5\mu_1 \quad (19)$$

$$\partial L/\partial\mu_1 = 0.75\chi_1 + 0.25\chi_0 - 0.2 = 0$$

$$\Leftrightarrow 0.75(0.5\mu_1) + 0.25(0.5\mu_1) - 0.2 = 0.5\mu_1 - 0.2 = 0$$

$$\Leftrightarrow \mu_1 = 0.4 \quad (20)$$

Substituting (20) into (18) and (19) yields the following values for χ_1 and χ_0:

$$\chi_1 = 0.2$$

$$\chi_0 = 0.2$$

In order to be a solution, these values must satisfy condition (15) of the Kuhn-Tucker conditions: $\partial L/\partial\mu_2 \geq 0$

$\partial L/\partial \mu_2 = 0.25\chi_1 - 0.25\chi_0 - 0.2 = 0.25(0.2) - 0.25(0.2) - 0.2 = -0.2 < 0$

Since these values give a negative $\partial L/\partial \mu_2$, they are not an optimal value set for the model. For similar, reasons, all other cases do not provide optimum values, and the calculations of each are omitted here.