# Simple Competitive Internet Pricing*

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**Abstract**

It is widely recognised that pricing is required to control congestion on the Internet. One lesson that has emerged from many proposals is that any price system should be simple and robust to competition. This is highlighted in the question currently under debate in the market for Internet services: should usage prices should be employed at all? In a duopoly model with overall positive network effects, it is shown that flat rate pricing can occur in equilibrium, even when the costs of measuring variable demand are very small.

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## 1. Introduction

By any measure, the growth of the Internet has been phenomenal: the number of hosts, the number of users, and the amount of traffic have been doubling approximately every year since 1988. The price of this success has been increasing congestion. Surfing the Web is notoriously slow during peak hours; by some estimates, 30% of Internet traffic is retransmissions of dropped packets.[1] There are two major causes of increasing congestion. The first is the appearance of new applications with high bandwidth requirements. A one minute Internet telephone call uses 500 times the capacity of a comparable paragraph of e-mail; one minute of video covering that same paragraph uses 15,000 times as much capacity, the equivalent of sending more than 10,000 pages of text. The second is increasing use of aggressive protocols to transport delay-intolerant traffic. TCP, the transmission control protocol of the Internet, adapts its sending rate to congestion conditions; in contrast, protocols such as UDP (user datagram protocol) used for e.g. voice telephony do not have such congestion avoidance behaviour.

It is widely recognised that pricing of Internet resources is required to control congestion. Economists argue that the large heterogeneity in uses of the Internet means that equal treatment of all sources (the current Internet practice) is inefficient. Network managers have concluded that increasing capacity without raising revenue is too expensive.[2] Pricing schemes have to balance (at least) two factors: simplicity and robustness to competition. This paper examines these two factors in a basic model of competition in non-linear prices with overall positive network effects; congestion is interpreted as a decrease in the level of positive effects. Section 2 considers briefly two Internet pricing proposals to illustrate the issues. Section 3 concentrates on the question currently being debated in the market—whether Internet usage should be charged at all. Section 4

[1]It is surprisingly difficult to obtain hard evidence of Internet congestion. See Paxson (1997) for an authoritative study of the area. Many university links to the public Internet are heavily loaded, which may be why academics think congestion is a problem. It may be, however, that the general problem is not congestion, but non-responding servers; see Huitema (1997).

[2]For example, the councils funding the U.K. academic network, JANET, decided to introduce volume-based pricing from August 1998 rather than continue to "provide 100% funding for what appears to be an unlimited demand for networking service" (JISC Circular 3/98, available at `http://bill.ja.net/`).

concludes. The appendix contains technical details.

## 2. Internet Pricing Proposals

A full survey of the many Internet pricing proposals is not possible here; see Gupta, Stahl and Whinston (1997), Kelly, Maulloo and Tan (1998), Mackie-Mason and Varian (1997) and Odlyzko (1997) for a few examples. This section looks at just two—the 'smart market' and 'Paris Metro Pricing'—in order to highlight two key issues for Internet pricing proposals. First, they must be simple enough to implement at reasonable cost. Secondly, they must be viable in a competitive setting.

The 'smart market' of Mackie-Mason and Varian (1997) is perhaps the best known of the Internet pricing proposals. It involves a zero usage price when network resources are not congested. At congested parts of the network, packets are prioritized according to bids attached to them by users; users whose packets are transmitted are not charged the amount that they bid, but rather the bid of the highest priority packet that is not transmitted. This Vickrey auction has well-known properties of incentive compatibility and efficiency.

There are several criticisms of this scheme. The first is that it fails to take into account the fact that users are interested not only in instantaneous resource allocation, but allocation over time (e.g. the duration of an Internet telephone call); for recent work on this question, see Crémer and Hariton (1999). Secondly, the smart market is generally viewed as being too complex to implement. The requirement that a bid is attached to every packet imposes large burdens on both users and already-congested resources (especially routers).[3]

Until 15 years ago, users of the Paris Metro were offered a choice of travelling in first or second class carriages. The only difference between the two carriages was the price

[3]The average packet size in TCP/IP, the suite of transport and application protocols used widely on the Internet, is 1,600 bits. So a short e-mail generates two or three packets; the postscript version of this paper requires over 200 packets for file transfer; a five minute telephone conversation generates around 1,500 packets.

charged: both carriages had the same number and quality of seats, and (obviously) both reached the destination at the same time. The first class carriage was, however, more expensive, and consequently (on average) had fewer passengers in it. Those users with a strong preference for e.g. obtaining a seat were willing to pay the higher price; others, content to travel in a more congested carriage (on average), paid the lower second class fare.

Odlyzko (1997) has proposed to apply the same scheme to the Internet. In the proposal, networks are partitioned into separate logical networks, with different usage charges applied on each sub-network. No guarantees of service quality are offered; but on average, networks charging higher prices are less congested. Users sort themselves according to their preferences for congestion and the prices charged on the sub-networks. The attraction of this scheme is its simplicity. Gibbens, Mason and Steinberg (1998) show, however, that PMP may not survive in non-competitive equilibrium. (Their analytical findings are similar to the numerical results of Wilson (1989), who shows the same for priority supply classes of electricity.)

## 3. Flat Rate v. Usage Pricing

The various economic and engineering solutions to Internet congestion are far removed from current practice. The central question for Internet Service Providers (ISPs) is not what auction scheme should be used, but rather whether usage prices should be employed at all. The emerging market consensus for dial-up access to the Internet is a flat rate pricing structure—unlimited access for a fixed monthly fee. Leased-line access is charged according to the capacity of the line. The majority of users therefore face a marginal usage price of zero.[4]

[4]There are, however, examples of usage pricing on the Internet. Users of NZGate, New Zealand's Internet gateway managed by Waikato University, are charged according to the volume of their traffic. Brownlee (1997) discusses the system, noting that the overheads of charging are significant. JANET, the U.K. academic network, charges users for incoming transatlantic traffic. The configuration of JANET means that measurement of traffic to and from North America is relatively inexpensive. Zero usage prices are more firmly established in the U.S..

Traditional explanations of this 'buffet pricing'—charging a fixed fee to allow unlimited consumption (often during a specified time period)—have focussed on pure cost (see e.g. Nahata, Ostaszewski and Sahoo (1999)) or uncertainty (see e.g. Fishburn, Odlyzko and Siders (1997)) factors.[5] This section concentrates on strategic reasons. A monopolist (weakly) prefers to use a non-linear pricing scheme. But duopolists must take into account whether non-linear pricing intensifies or lessens competition. In the model here, two-part tariffs can cause duopolists to compete more fiercely than flat rates. As a result, even a very small cost of implementing a pricing scheme with a usage component can make a large difference to the equilibrium outcome.

### *3.1. A Simple Model*

This section develops a Hotelling-type model with variable demand and overall positive network effects to examine the possibility that flat rate pricing may occur in equilibrium.

Consumers are distributed uniformly along the unit interval. Each consumer has a linear demand for a firm's product: with a usage price of $p$ per unit, the consumer demands $1-p$ units of the product. There are two firms, labelled 0 and 1, located respectively at 0 and 1 on the line. A consumer located at $0 \leq x \leq 1$ receives a utility from buying from firm 0 of

$$U(x,0) = V + t(1-x) + \frac{1}{2}(1-p_0)^2 + nD_0 - f_0. \tag{1}$$

In equation (1), $V$ is a positive constant representing a common utility received by all consumers from either firm's product. The term $t(1-x)$ represents the transport cost element of the consumer's utility; $t \geq 0$ is a scaling parameter. $\frac{1}{2}(1-p_0)^2$ is the surplus gained by the consumer (ignoring income effects) from consumption of firm 0's product when the usage price is $p_0$. $nD_0 \geq 0$ is a network effect term, where $D_0$ is the total demand of consumers who buy from firm 0 and $n \in [0,t]$ is a constant. Finally, $f_0$ is the fixed price charged by firm 0.

[5] A full survey of buffet pricing is not possible here; see Nahata, Ostaszewski and Sahoo (1999) for many examples.

Note two things. First, the network effects depends only on firm 0's demand and not on firm 1's; in effect, the firm's products are assumed to be incompatible. At first, this may seem a strange assumption to make in the context of the Internet: after all, the Internet is nothing more than a 'network of networks', defined by widespread connectivity between multiple networks. As the nature of the Internet changes from its original academic origin to a commercial market place, however, the issue of interconnection agreements between asymmetric networks is becoming increasingly relevant. Since 1996, large ISPs (or backbones) have changed radically their interconnection terms, leading to claims that dominant networks are abusing their market positions by refusing or degrading the quality of interconnection with smaller networks. See Crémer, Rey and Tirole (1998) and Mason (1999) for discussion and analysis. In addition, if the firms' products in this model are interpreted as communication services (e.g. voice telephony or video conferencing), then lack of compatibility can arise through the use of proprietary standards in the communication applications. This paper does not analyse the compatibility question (see Katz and Shapiro (1985) and Mason (1999) for this). Instead, it assumes that compatibility is less than perfect; and to simplify matters, that there is complete incompatibility. Allowing for partial compatibility would produce no substantial change in the model's results, but only complicate the calculations.

Secondly, it is assumed that positive network effects dominate, so that $n \geq 0$. Congestion in this model is equivalent to a reduction in $n$, but with $n$ remaining non-negative. A more complete treatment of negative network effects would be an important extension of the model.

Similarly, a consumer located at $0 \leq x \leq 1$ receives a utility from buying from firm 1 of

$$U(x, 1) \quad = \quad V + tx + \frac{1}{2}(1 - p_1)^2 + nD_1 - f_1, \tag{2}$$

where the terms have the same interpretation, and firm 1 charges prices $(f_1, p_1)$.

The firms have equal production costs: a fixed cost $k \geq 0$ per customer; a (constant) cost $c \in [0, \tilde{c})$ per unit of demand ($\tilde{c}$ is defined in lemma A.2 in the appendix); and a fixed cost of $m$ if a pricing scheme with a usage component (e.g. a two-part tariff) is employed, but not if a flat rate scheme is used. The cost $m$ represents the cost of equipment to measure consumers' demand (e.g. counting packets on the Internet).

The following assumptions are made:

ASSUMPTION 1: *$V$ is sufficiently high that in equilibrium, all consumers buy from one of the firms; but $V$ is not so high that any consumer would wish to buy from both firms.*

ASSUMPTION 2: *The firms' pricing strategies are restricted to flat rates or two-part tariffs.*

Assumption 2 is by far the more restrictive. It is used for pragmatic reasons: the limited ambition of this section is to consider whether it is possible for equilibrium to involve only volume-independent prices. There are two aspects to the restriction. First, even when prices do not depend on usage, they may nevertheless be non-linear; for example, current Internet pricing schemes rarely depend on volume, but often are non-linear in the maximum bandwidth available. Secondly, a full analysis would compare volume-dependent and -independent pricing schemes more generally; see e.g. Stole (1995), Armstrong and Vickers (1998), and Rochet and Stole (1999). This section considers only the simplest cases of both schemes.

The game has two stages. In the first stage, the firms choose what type of pricing scheme to employ (a flat rate or a two-part tariff) and the level of price(s) simultaneously.[6] In the second stage, consumers choose which firm to buy from and how much to buy. The analysis concentrates on symmetric Nash equilibria in pure strategies.

[6]The results are not sensitive to this extensive form; in fact, they are strengthened if the firms choose the type of pricing scheme before the price levels.

The main result of this section is illustrated in figure 1, which shows the regions in $(m, n)$ space in which different equilibrium outcomes hold.[7] The full details are stated in proposition 1 below. The two-part tariff solution can be an equilibrium only if the fixed cost of charging usage prices is not too large: $m < \hat{m}$. Conversely, the flat rate solution can be an equilibrium only if the fixed cost of charging usage prices is sufficiently large $(m > \tilde{m})$.[8] In particular, the flat rate equilibrium does not exist when $m = 0$. Lemma A.2 in the appendix shows that in this case a two-part tariff is always a best response to the other firm charging a flat rate, and consequently there cannot be a flat rate equilibrium.

The figure also indicates that there may be zero, one or two possible equilibria, depending on the values of $m$ and $n$, and the various critical levels. When an equilibrium exists, the flat rate is the unique equilibrium for 'high' values of $m$ (greater than $\hat{m}$), and the two-part tariff the unique equilibrium for 'low' values of $m$ (lower than $\tilde{m}$). For 'intermediate' values of $m$ (between $\tilde{m}$ and $\hat{m}$), multiple equilibria can arise for sufficiently low $n$, while no equilibrium exists for sufficiently large $n$. Note, however, that the figure shows only the case in which $\hat{m} > \tilde{m}$. It is possible that $\hat{m} < \tilde{m}$, in which case multiple equilibria do not arise.

The following proposition, proved in the appendix, states the general conditions for existence of symmetric equilibrium. (The equilibrium prices are given below, while the various critical values of $n$ and $m$ are defined in the appendix.)

PROPOSITION 1: *There exists an equilibrium in which both firms choose a two-part tariff if and only if $n \leq n^*$ and $m \leq \hat{m}$.*

*When $n \leq \tilde{n}$, there exists an equilibrium in which both firms choose a flat rate if and only if $m > \tilde{m}$. When $n \in (\tilde{n}, \overline{n}]$, the necessary and sufficient condition is $m > \overline{m} > \tilde{m}$. When $n > \overline{n}$, $m > \overline{m}$ is sufficient but not necessary.*

[7]In fact, the figure simplifies the situation substantially. Nevertheless, the figure shows some of the general features of the proposition.

[8]This is true for the example illustrated in the figure; proposition 1 gives the general condition.

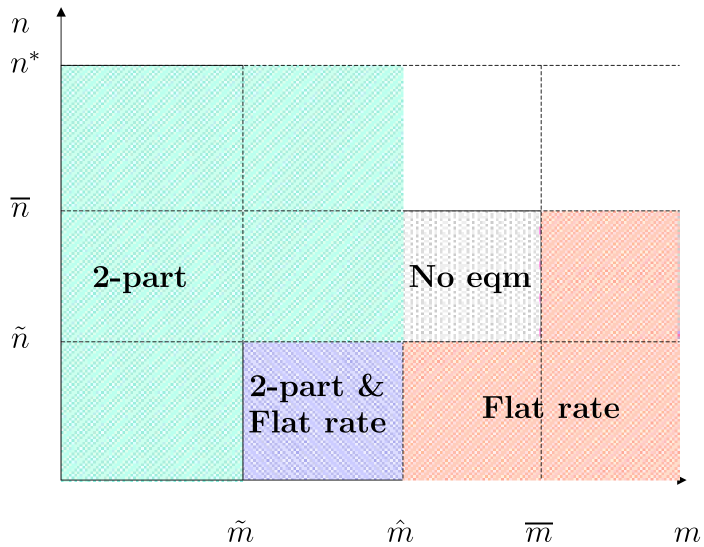


Figure 1: Equilibrium Existence

Recall that, in this model, congestion is interpreted as a decrease in $n$. The proposition indicates, therefore, that flat rate pricing can occur in equilibrium between two firms subject to overall positive network effects but with congestion. The value of $m$ sufficient to ensure existence of the flat rate equilibrium can be very small. To illustrate this, suppose that $t = 1, c = 0.04$ and $m = 0.003$.[9] Then for $n < 0.3026$, the unique symmetric Nash equilibrium involves a flat rate. In this case, a fixed cost of measuring demand that is an order of magnitude less than unit cost ensures that no usage pricing occurs in equilibrium.

In the symmetric two-part tariff and flat rate equilibria analysed in the proposition,

$$f_{\text{2part}} = t + k - \frac{n}{2}\left(1 + \frac{n}{2} - c\right), \qquad p_{\text{2part}} = c - \frac{n}{2}; \tag{3}$$

$$f_{\text{flat}} = t + k + c - n. \tag{4}$$

The usage price $p_{\text{2part}}$ in the two-part tariff is less than unit cost $c$; the fixed price $f_{\text{2part}}$

[9]In this case, $\tilde{n} = 0.6803$, $\hat{n} = 0.7370$, $\overline{n} = 0.7406$, and $n^* = 0.7492$; and $\tilde{m} = 4.8 \times 10^{-6}$ and $\overline{m} = 0.1157$.

is below the standard Hotelling level, $t+k$. Positive network effects therefore decrease both prices: the firms compete more fiercely in order to increase their market share and total demand, and so be more attractive to consumers. For sufficiently large $n$, greater than $n^*$, equilibrium in two-part tariffs does not exist. Note the usage price $p_{2\text{part}}$ in the two-part tariff can be negative: for $n > 2c$, the firms subsidise demand for their products. The flat rate $f_{\text{flat}}$ has a similar interpretation: it is the Hotelling price (unit production cost $k+c$ plus horizontal mark-up $t$) minus an amount to reflect increased competition when positive network effects are present.

Which equilibrium yields the greater profit depends on the intensity of competition for the marginal consumer, and the size of the fixed cost $m$. In the two-part tariff equilibrium, $\pi_{2\text{part}} = \frac{1}{2}(t-n) - \frac{n}{2}\left(\frac{n}{2}-c\right) - m$; in the flat rate equilibrium, $\pi_{\text{flat}} = \frac{1}{2}(t-n)$. Suppose that $m = 0$, so that only the competitive effect remains. When $n = 0$, the profits in both equilibria are equal to $\frac{t}{2}$. (In the flat rate equilibrium, all consumers demand 1 unit, and so the model is identical to the usual Hotelling set-up. In the two-part tariff equilibrium, the usage price is set at cost and standard Hotelling profits are earned on the fixed charge.) When $n$ is greater than zero but less than $2c$, the firms enjoy two benefits in the two-part tariff equilibrium relative to the flat rate: they make a smaller loss on variable demand (since the usage price is positive); and they compete less fiercely for the marginal consumer, since she receives (all other things equal) a smaller surplus when the usage price is positive. As $n$ grows larger, however, the usage price decreases until, at $n = 2c$, prices in the two equilibria are equal: both involve a zero usage price and a fixed price of $t+k-c$. From this point onwards ($n > 2c$), the benefits to the firms of the two-part equilibrium relative to the flat rate are reversed: now they make a greater loss on variable demand, and they compete more fiercely for the greater surplus of the marginal consumer.

### *3.3. Welfare Comparisons*

It is straightforward to calculate the social surplus in the two equilibria. The various expressions (which are omitted here) lead directly to the following proposition:

Proposition 2: *Social surplus is greater (less) with the two-part tariff equilibrium prices than the flat rate equilibrium prices iff* $(n-2c)^2 > (<) 16m$.

(Note that the proposition does not depend on whether a particular equilibrium exists.) It is intuitive that a small fixed cost of charging usage prices $m$ favours the two-part tariff equilibrium. Social surplus is also higher in this equilibrium when network effects are either small or large. From the previous section, profits are lower in the two-part tariff equilibrium whenever $n$ is low or high, when $m$ is strictly positive. For sufficiently small and sufficiently large network effects, consumer surplus is higher under two-part pricing; for intermediate levels, consumer surplus is higher with flat rate pricing. To understand this, start at the extreme of $n = 0$. Proposition 1 shows that in this case, $f_{\text{2part}} = t + k$ and $p_{\text{2part}} = c$ and $f_{\text{flat}} = t + k + c$. Consumers demand less under the two-part tariff, but also pay less. The reduction in price paid is first order, while the reduction in surplus is second order. Hence consumer surplus is higher with the two-part tariff. By continuity, for $n$ close to 0, consumer surplus is higher under the two-part tariff. Now consider the opposite extreme: $n \geq 2c$. Proposition 1 shows that $f_{\text{2part}} \leq f_{\text{flat}}$ and $p_{\text{2part}} \leq 0$, so that clearly consumer surplus is greater in the two-part tariff equilibrium. For intermediate values of $n$, the reduction in demand from a positive usage charge means that the two-part tariff yields lower consumer surplus than the flat rate. When $n$ is sufficiently small or large (so that $(n-2c)^2 > 16m$), the combined effect of a greater consumer surplus and a lower profit is such that social surplus is higher in the two-part tariff equilibrium.[10]

## 4. Conclusions

One lesson that has emerged from the many proposals for Internet pricing is the need for simple schemes that are robust to competition. This paper has considered whether flat rate pricing (currently the consensus for Internet charging) has an economic foundation in a model of competition between differentiated firms in the presence of overall positive network effects, in which congestion is interpreted as a reduction in the size of these

[10] Social surplus is decreased by imposing an upper bound on the usage price that can be charged by the firms. I am grateful to the referee for raising this question.

positive effects. It has shown that even a very small fixed cost of implementing usage prices can lead to situations in which only flat rate pricing survives in equilibrium.

Space constraints have prevented consideration of extensions to the analysis. More general network effects could be considered, including a more complete treatment of congestion. The restriction on firms' pricing strategies should be relaxed to allow full nonlinear pricing (rather than only two-part tariffs). These and other changes raise interesting issues for further research.

## APPENDIX

This appendix contains the proof of proposition 1. The derivations of the two-part tariff $(f_{\text{2part}}, p_{\text{2part}})$ and the flat rate $f_{\text{flat}}$ equilibrium prices in equations (3) and (4) are completely standard for Hotelling models and so are omitted.

Consider first the case of one firm deviating to a flat rate scheme while the other charges the two-part tariff $(f_{\text{2part}}, p_{\text{2part}})$.

Lemma A.1: *A necessary and sufficient condition for a unilateral deviation to a flat rate from the two-part tariff* $(f_{\text{2part}}, p_{\text{2part}})$ *not to be profitable is* $m < \hat{m}$*; a necessary condition is* $n < \hat{n}$*, where* $\hat{m} \equiv \frac{1}{128}\left(\frac{\left(4(8t-c^2)-17n^2-4(8-7c)n\right)(n-2c)^2}{4(t-n)-n(n-2c)}\right)$ *and* $\hat{n} \equiv \frac{2}{17}(2\sqrt{16-28c+8c^2+34t} - (8-7c)) < n^* \equiv -(1-c)+\sqrt{(1-c)^2+2t}$.

Proof: The first-order condition for the profit-maximising choice of a flat rate as a best response to the two-part tariff $(f_{\text{2part}}, p_{\text{2part}})$ gives $f_D = t+k+c-n-\frac{1}{16}(7n-2c)(n-2c)$. The second-order condition is satisfied. The profit earned by the deviating firm is $\pi_D = \frac{1}{128}\left(\frac{(16(t-n)-(7n-2c)(n-2c))^2}{4(t-n)-n(n-2c)}\right)$, and so the additional profit earned by the deviating firm is $\Delta_D \equiv \pi_D - \pi_{\text{2part}} = \frac{1}{128}\left(\frac{(17n^2-28nc+4c^2-32(t-n))(n-2c)^2}{4(t-n)-n(n-2c)}\right) + m$. The expression for $\Delta_D$ gives the results in the lemma. Notice that since the definition of $\hat{m}$ involves a quartic in $n$, there is another region, $n > -(2-c)+\sqrt{(2-c)^2+2t} > \hat{n}$, in which $\hat{m} > 0$. But if $c \leq \frac{t}{2}$, this region is ruled out by the condition that $n < n^*$, as $-(2-c)+\sqrt{(2-c)^2+2t} > n^*$ in this case. □

Consider the other case, where one firm deviates to a two-part tariff when the other charges the flat rate $f_{\text{flat}}$.

Lemma A.2: *When* $n \leq \tilde{n}$*, a necessary and sufficient condition for a unilateral deviation to a two-part tariff from the flat rate* $f_{\text{flat}}$ *not to be profitable is* $m > \tilde{m} \equiv \frac{2c^3}{27t}$*. When* $\tilde{n} < n \leq \overline{n} \equiv \frac{-2(2-c)+\sqrt{(2-c)^2+8t}}{2}$*, a necessary and sufficient condition is* $m > \overline{m} \equiv \frac{1}{4}\left(2(1+t)+c(1+c) - (1+c)\sqrt{(2-c)^2+8t}\right)$*. When* $\overline{n} < n \leq t$*,* $m > \overline{m}$ *is sufficient for the deviation not to be profitable.* $\tilde{n}$ *is the smallest root lying in the interval* $[0,t]$ *of the cubic equation in n:* $12\tilde{n}^3 + (16(1-c)-12t+c^2)\tilde{n}^2 - 16(2-c)t\tilde{n} + 16t^2 = 0$.

Proof: When $n \leq \tilde{n}$, the best two-part response by a firm when the other firm charges the flat rate $f_{\text{flat}}$ involves both firms having a positive market share. The best response tariff is

$f_I = \frac{8t^2-(2c-3k-4)n^2-4(3-c)tn-(2t-n)\sqrt{\phi}}{3n^2}$ and $p_I = \frac{-4(t-n)+nc+\sqrt{\phi}}{3n^2}$, where $\phi \equiv 12n^3 + (16(1-c) - 12t + c^2)n^2 - 16(2-c)tn + 16t^2$. The deviating firm earns a profit $\pi_I$. The difference between $\pi_I$ and $\pi_{\text{flat}} = \frac{1}{2}(t-n)$ is bounded above: $\Delta_I \equiv \pi_I - \pi_{\text{flat}} \leq \frac{2c^3}{27t} - m$. The value $\tilde{n}$ is such that $n \leq \tilde{n}$ ensures that $\phi \geq 0$. For $c$ less than some value $\tilde{c} < \frac{t}{2}$, the root exists.

When $\tilde{n} < n \leq t$, the best two-part response by a firm when the other firm charges the flat rate $f_{\text{flat}}$ involves the deviating firm capturing the entire market. Suppose that firm 1 deviates, while firm 0 charges the flat rate. Let firm 1's best response tariff be $(f_B, p_B)$. Let the consumer who is indifferent between firm 0 and firm 1 be located, in general, at $x^*$. Firm 1's profit is $\pi_1 = (f_B - k) + (p_B - c)(1 - p_B)$, since firm 1's market share is 1 ($x^* = 0$) in this case. The indifference relation that determines $x^*$ can be manipulated to give $\pi_1 = -\left(\frac{p_B^2}{2} + (n-c)p_B\right)$. Choose $p_B$ to maximise $\pi_1$ i.e. $p_B = -(n-c)$. This implies, via the indifference relation, that $f_B = k + n + \frac{1}{2}(3n-c)(n-c)$. Therefore firm 1 earns a profit of $\pi_B = \frac{(n-c)^2}{2} - m$. In order for these prices to be consistent with profit maximisation, it must be that $\frac{\partial \pi_1}{\partial p_B}$ and $\frac{\partial \pi_1}{\partial f_B}$ are both less than zero; and $\frac{\partial x^*}{\partial p_B}$ and $\frac{\partial x^*}{\partial f_B}$ are greater than zero.[11] It is straightforward, but lengthy to show that provided $n < \overline{n} = \frac{-(2-c)+\sqrt{(2-c)^2+8t}}{2}$, these inequalities are satisfied. When $\overline{n} < n \leq t$, the prices $(f_B, p_B)$ are not feasible, and the deviation profit is lower than $\pi_B$. The difference between $\pi_B$ and $\pi_{\text{flat}}$ is $\Delta_B \equiv \pi_B - \pi_{\text{flat}} = \frac{(n-c)^2}{2} - \frac{t-n}{2} - m$. Since $\Delta_B$ is increasing in $n$, it reaches a maximum at the upper bound of $n = \overline{n}$. Therefore $\Delta_B \leq \frac{1}{4}\left(2(1+t) + c(1+c) - (1+c)\sqrt{(2-c)^2+8t}\right) - m$. Hence if $\tilde{n} < n \leq \overline{n}$, then a necessary and sufficient condition for the deviation not to be profitable is that $m > \underline{m} \equiv \frac{1}{4}\left(2(1+t) + c(1+c) - (1+c)\sqrt{(2-c)^2+8t}\right)$. If $\overline{n} < n \leq t$, $m > \underline{m}$ is a sufficient, but not necessary, condition for the deviation not to be profitable. □

[11] The converse would also do i.e. $\frac{\partial \pi_1}{\partial p_B} \geq 0$, $\frac{\partial x^*}{\partial p_B} \leq 0$, etc.. It turns out that this case is not relevant.

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